A THEORY OF RELEVANT PROPERTIES 1: REFLECTIONS AND DEFINITIONS

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ABSTRACT: In the paper a theory of relevant properties is developed. The theory permits us to distinguish between properties that are relevant to an object and the properties that are irrelevant to it. Predication is meaningful only if a property is relevant to an object. On the base of introducing a special negative type of predication as opposed to usual sentential negation, a new notion of generalization for properties is defined. Context-free, as well as context-depended relevance of properties are considered.

Keywords: objects, properties, relations, particulars vs. universals, generalization of properties, natural kinds, relevance of properties.

CONTENTS

1. Preliminaries

There are many ways to classify properties. For example, Locke's division between primary and secondary qualities is widely known. Another possible classification that can be found in philosophical literature is between necessary (or essential) and accidentally properties, as well as between intrinsic and external ones. There are, however, a number of cases, when the question, whether a property is necessary (essential) or accidental (nonessential) to an object, is simply *premature*. The classical examples are those of Caesar or of the moon, if one asks:

(1) Is Caesar prime (a prime number)?
(2) Is the moon honest?

I propose to explain such cases in terms of *relevance-irrelevance*. It is clear that the property "prime" *has nothing to do* with Caesar, i.e., it is *irrelevant* to him. And so is the property "honest" to the moon. The property "prime", however, is relevant to *any* natural number (no matter whether a certain number is prime or not). And so is the property "honest" to every human being.

Let us divide all the properties (with respect to some object) on such that are relevant to the object, and such that are irrelevant to it. Considering the properties that are relevant to an object, we may notice that the object instantiates some of them and does not instantiate others. As to irrelevant properties, the situation is quite different -it is impossible to imagine that the object has any of these properties at all: it simply *cannot* instantiate such properties. Thus, the following conjecture looks quite natural:

(RP) *An object can instantiate a property iff the property is relevant to the object.*

It can be shown that many paradoxes known in philosophy of science and semantics of natural language arise from an attempt to ascribe an irrelevant property to an object. The present paper is a part of a general project devoted to the phenomenon of relevance with respect to properties. The aim of the whole project is to develop a formal theory of properties that takes (RP) as one of the main principles governing predication, and to explicate on this way our natural language intuitions on the relevance of properties to objects. The theory may have various applications: in solving logical and semantical paradoxes, in elucidating the relations between particulars and universals, in argumentation theory, etc. In this paper we present general reflections on the subject and propose some key definitions. The applications will be developed in further papers.

2. Dunn on relevant predication. Relevant predication vs. relevant properties

Dunn (1987) proposes a formal theory of relevant predication (reproduced with small changes in (Anderson, Belnap and Dunn 1992, pp. 445-472). He considers the difference between predicating a property of "being such an *x* that Socrates is wise" to Socrates himself and ascribing it to Alcibiades. As a result, one obtains the following two propositions:

(3) Socrates is such that he is wise.
(4) Alcibiades is such that Socrates is wise.

Whereas the case (3) represents quite a "normal" (relevant) predication, (4) -Dunn states- is a glaring example of an "irrelevant predication".

Dunn's analysis essentially rests on some principal notions of λ-calculus. Using lambda abstraction, any formula Fx can be transformed into a predicate λxFx that represents "the property of being (an x such that x is) F". Dunn also makes use of the principle of β-conversion:

(β) $(\lambda xFx)a \Leftrightarrow Fa.$

Let a stand for Socrates and F stand for the property "wise". Then λxFx means *the property of being an x such that x is wise*, and $(\lambda xFx)a$ means *Socrates is such that he is wise*.

In general, however, lambda-abstraction can be applied even to formulas which do not contain free occurrences of x. The corresponding lambda-expression is λxA (where A is a sentence) that represents "the property ascribed to x is saying that A". As a result of applying β-conversion to closed formulas one obtains the formula itself:

(β) $(\lambda xA)a \Leftrightarrow A.$

Now let *a* stand for Alcibiades and *A* stand for the sentence "Socrates is wise". Then *(λxA)a* means simply *Alcibiades is such that Socrates is wise*.

Dunn remarks that classical lambda calculus gives no means for distinguishing between (3) and (4). Moreover, he shows that validity of (4) essentially depends on the so-called *Positive Paradox of Relevance*:

(PP) A 6 $B \rightarrow A$,

whereas the validity of (3) does not. Indeed, there is a strict analogy between (3) and (4) and the following statements:

(3') If anyone is Socrates, then it is wise.
(4') If anyone is Alcibiades, then Socrates is wise.

We obtain (3') by means of the following argument:

(3'') Socrates is wise. Therefore, if *x* = Socrates, then *x* is wise,

which is a simple instance of *Indiscernability of Identicals*:

(II) Fa 6 $x=a \rightarrow Fx$.

But the corresponding argument for (4') is a clear instance of (PP):

(4'') Socrates is wise. Therefore, if *x*=Alcibiades, then Socrates is wise.

Dunn points out that an attempt to distinguish between (3) and (4) by means of a restriction saying that formation of a lambda expression *λxA* is allowed only when *A actually* has at least one free occurrence of the variable *x* fails, because of equivalencies such as:

(&1) $A \Leftrightarrow A$ & $(Fx \vee \sim Fx)$,
(&2) $A \Leftrightarrow A$ & $(A \vee Fx)$.

Dunn concludes that we need a special definition of *relevant predication.* He considers the famous statement of Juliet (or Shakespeare):

(5) "A rose by any other name would smell as sweet",

and qualifies "sweet smell" as a relevant property of a rose. Taking (5) as a paradigmatic case of relevant predication, he proposes the following definition:

Definition 1. $(\rho x Ax)a \Leftrightarrow \forall x\ (x=a \rightarrow Ax)$. (The expression "$(\rho x Ax)a$" is read as "*a* relevantly has the property of being (an x) such that *A*".)

This definition works quite well in many cases. However, the notion of the relevance that it produces is not the notion we are looking for, and it does not reflect the intuition outlined in the preliminary section. This definition cannot reflect all the cases when a property could be relevant to an object. For example, one of the consequences of definition 1 is that if a property holds relevantly of an individual, then it also just plain holds of the individual (see fact 2 from Dunn 1987). But we are prone to say that a property can be relevant to an individual without holding of it. What Dunn's definition really defines, we believe, is the "non-fictional" (or *real*) predication. This is a very useful and heuristically valuable notion, but we still need to find a suitable definition that could explain us what does it mean that a property is relevant to an object. The difference we wish to explicate is the difference between relevant *predication* and the *relevance of properties*. Moreover, as will be shown below, there are contexts, where the property of being such (an *x*) that Socrates is wise can be considered as *relevant* to Alcibiades. The above definition does not allow to explain such contexts as well.

Dunn introduces also the notion "φx is a formula of a kind that determines relevant properties (with respect to *x*)" as follows:

Definition 2. "φx is a formula of a kind that determines relevant properties (with respect to x)" $\Leftrightarrow$ $\forall x\ (\varphi x \rightarrow \forall y\ (y=x \rightarrow \varphi y))$, where y is not free in φ.

This definition also does not give us an idea of the relevance that we wish to obtain. We expect to have a definition that can tell us when a property is relevant to a concrete individual. If we consider a particular instance of the right-hand side of the definition 2:

$$Fa \rightarrow \forall y\ (y=a \rightarrow Fy),$$

we may notice that it says us something about a relation between F and a, when a instantiates F, but it does not say us anything, if a does not instantiates F. This is rather a definition of some sort of *strict* occurrence of a variable x in the formula φx, and Dunn indeed considers in his paper such an interpretation of the definition 2. But the problem of relevance of a property to an object still has to be solved.

3. The ways of speaking about properties

To begin with it is appropriate to make some remarks about certain important principles of the further analysis. The "canonical" way of representing properties within various theories of properties (such as in Feferman 1984, Turner 1987, Bealer and Mönich 1989, Kamareddine 1992) is to use λ-operator and (β)-conversion (or their analogues). Nevertheless, this apparatus taken unrestrictedly can produce some unwelcome consequences. As is very well-known, (β) together with simple logical principles easily leads to a contradiction (Russell's paradox). Let us define F as $\lambda x.{\sim}xx$. Then we have:

1. $(\lambda x.{\sim}xx)F \Leftrightarrow {\sim}FF$	(β)
2. $FF \Leftrightarrow (\lambda x.{\sim}xx)F$	(definition of F)
3. $FF \Leftrightarrow {\sim}FF$	(1, 2)

As Kamareddine (1992, pp. 79-80) points out, there are two main elimination strategies for Russell's paradox that are represented by two different routs of research. In line with the first strategy one proceeds from the assumption that the contradiction is caused by the principle of *self-application.* Therefore, one can avoid the paradox, by introducing a *typed* theory of some sort. The point of any typed theory consists in restricting just the above mentioned principle. Russell himself was the first who proposed to resolve the paradox on this way -by means of the typed set-theory. The typed λ-calculus also belongs to this rout of research. This approach, however, is not very popular, as typed theories are quite cumbrous and inconvenient to use.

Therefore, the second strategy could seem to be more attractive (as many authors believe), according to which one restricts logic, but keeps type-freenes and saves the principle of self-application. In case of properties this means an acceptance of a general principle of *self-predication*. This principle constitutes an important desideratum of many property theories. As Chierchia and Turner put it:

(SP) "Properties can be truly predicated of themselves" (Chierchia and Turner 1988, p. 263).

Oliver expresses the same idea on a "metaphysical" level:

> Particulars have or instantiate properties but not vice versa. Properties may themselves have or instantiate properties, but do not have or instantiate particulars (Oliver 1996, p. 20).

Nevertheless, the above mentioned strategies are not the only possible ones. For example Wessel (1995) presents a *theory of terms*, where a strong division between *subject terms* and *predicate terms* is realized. Subject terms should denote objects, and predicate terms serve for expressing properties and relations. One of the main principles of the theory is

(TT) "No subject term is a predicate term, and no predicate term is a subject term" (Wessel 1995, p. 356).

On the one hand, this principle entirely locks out Russell's paradox. On the other hand, we still need some tools to be able to state something about properties themselves. Consider the following examples:

(6) Socrates is wise.
(7) Socrates is such that he is wise.
(8) The property "wise" (the property of wisdom) is worthy of respect.

Statements (6) and (7) express essentially the same idea, although by different language means. The case (8) is of completely another character. The point is that whereas in (6) and (7) a property ("wise") takes a predicate position, in (8) the same property plays the role of a subject. It can seem that (8) is incompatible with (TT). However, it is possible to save and explicate *both* -(TT) and (8)- by means of introducing some *term-building* operators (cf. also Scheffler 1998).

The language we use contains individual variables, singular subject terms and predicate terms as primitive symbols. Wessel introduces also *general subject terms* as a special syntactic category as opposed to the "real" predicate terms. Such a distinction can be of great importance for some purposes of logical analysis of natural language. However, in the theory developed in the present paper it is not essential, so we do not introduce special general (subject) terms in our language.

Use x, y, z, x_1,... to range over individual variables; a, b, c, a_1,... to range over singular subject terms and P, Q, R, P_1,... to range over predicate terms. A general presupposition is that all the terms are non-empty. Another important principle that we adopt in our theory is (TT). Predicate terms can be *ascribed* (applied) to subject terms to get sentences. Sometimes we will use a "traditional" notation with brackets -$P(a)$, and sometimes the brackets will be omitted. We have also usual quantifiers and Boolean conjunction, disjunction and negation. The connection of implication will play an important role in our theory (as it usually does in many theories). Thus, we will need a "real" implication to avoid very well-known difficulties and paradoxes of material implication (especially those that are connected with fallacies of relevance -it would be of course highly unwelcome to have these fallacies in a theory that claims to explain the phenomenon of relevant properties). As the theory of relevant implication seems to be the most carefully elaborated theory among modern theories of "non-paradoxical" implication, we (following Dunn) employ the system of relevance logic *RQ* with identity. This adds the connective of relevant implication $\rightarrow$ to our language.

If *Ax* is a language expression having (perhaps) free occurrences of *x*, then *x.Ax* is a term (to be read as "an *x* such that *Ax*"). Let us introduce also two term-forming operators: [ ] -for creating predicate terms from subject terms, and π -for creating subject terms from predicate terms. The rules are as follows:

(PT) If *x.Ax* is a subject term, then [*x.Ax*] is a predicate term.
(ST) If *P* is a predicate term, then πP is a subject term.

The reading for [*x.Ax*] is -"the property of being (an *x* such that *x* is) *A*", and for πP- "the property *P*". That is, having *one and the same property*, we distinct two *modes of speech* about it. One should notice, that a property, beeing expressing in the form πP, can be used only in subject position, and this is *the only* possibility to put it in this position.

Now let *a* stand for Socrates, *W* -for "wise" and *R*- for "worthy of respect". Then formalizations for (6)-(8) are as follows:

(6') *Wa*
(7') [*x.Wx*]
(8') $R(\pi W)$

It is easy to see that now Russell's paradox cannot even be formulated, because *FF* is simply not well-formed expression of the language. However, our theory (as well as the one of Wessel's) is not a real (full) typed theory, because we do not introduce types for each level of language expressions. The only thing we do -we strongly distinguish between occurrences of a property in a subject and in a predicate position. This solution can be considered as a restricted typed theory. It allows only *two* types of terms, but does not involve (as usual type-theories do) types for more than the second level.

The acceptance of (TT) has some important ontological consequences -it results in a kind of *hierarchical ontology*. It means a strong ontological division between particulars and universals -no particular is an universal and *visa versa*. Thus, we take for granted that *only* particulars can instantiate properties, and do not agree with the above statement of Oliver. On the contrary, we believe that properties cannot *really* instantiate properties. It does not mean, however, that statements like (8) are illegal. We have to distinguish between what is going on in our language and in reality. There is no one-to-one correspondence between the (non-linguistic) world and the language: the latter is more complex. This complexity is introduced by people. People often handle some phenomena -which originally are not things at all-, as if they were a sort of things: the objects of their consideration. This happens, e.g., when we consider properties, we transform them in some kind of "objects" and obtain thereby a possibility to characterize them in that or another way. However, after a property becomes an object of our consideration, it does not become a real particular (or individual)! The formal apparatus outlined above makes it possible to combine the ontological rigor with the flexibility of our language -we can *speak* about properties in the "subject mode" without any commitment to regard them as real particulars.

4. Negative predication

A special type of predication plays a key role in our theory, namely the negative one, that can be introduced side by side with ordinary (positive) predication. As Fuhrman (1998) points out, it was a long tradition in logic to distinguish between negation of a sentence and negation of a predicate. This tradition seems to be almost forgotten (as we believe -undeservedly) in many of current logical works. This is all the more strange as one of the founders of the modern symbolic logic, Charles Peirce, in his article for the Baldwin's Dictionary of Philosophy and Psychology (1902) clearly distinguished between negation "as denying the proposition" and negation "as denying the predicate". Some modern authors also use the difference between these two kinds of negation, as, e.g. Routley (1980), "exploring Meinongs's jungle", does. R. Turner (1987) also employs "two negations which are both classical but are not interdefinable" (Turner 1987, p. 459). He points out that "internal negation of properties is not always equivalent to sentential negation" and would like to "institutionalize this by introducing 'negative' predication relations" (ibid.).

In the 60-70th Sinowjew and Wessel carefully elaborated and axiomatized a special predication theory that deals with both types of predication (see, e.g., Wessel 1998). They called it *the non-traditional theory of predication*. The crux of the negative predication consists in that it expresses a special type of relation, namely the denying ("*absprechen*") relation between predicate and subject, as opposed to a statement that it is not the case that a predicate is ascribed to an object. Thus, statements like "It is not the case that Socrates is wise" and "Socrates is not wise" generally are not equivalent. The difference may be not so evident in the example of Socrates, but it becomes obvious, if we compare the statements "It is not the case that number 2 is wise" and "Number 2 is not wise". The first sentence is true, whereas the second one is false (as well as "Number 2 is wise").

Let us introduce a special *-operation that represent the negative predication. That is, P^*a means that predicate P is "internally" denied of a. It is important that P^*a represents a simple sentence, as well as Pa. P^*a and $\sim Pa$ are generally not equivalent, only the following holds:

(NE) $P^*a \rightarrow \sim Pa$ (see Wessel 1998, p. 164).

(NE) is the only axiom that has to be added to predicate logic for axiomatizing negative predication. The analogue for the law of excluded middle ($P^*a \vee Pa$) does not hold for negative predication.

5. Simple predicates. A generalization of properties

At first, we confine ourselves with the *simple* predicates. We adopt the following version of (β) (which we call "the scheme of abstraction"):

(AB) $[x.Fx]a \leftrightarrow Fa$.

The direction from left to the right is called *Reduction*, and from right to the left -*Expansion* (cf. Turner 1987, p. 459). We call a predicate *simple* iff an application of Reduction to it results in a simple sentence. Simple predicates stand for simple properties. We introduce also the following scheme of abstraction for negative predication (for simple predicates):

(NA1) $[x.Fx]^*a \leftrightarrow F^*a$.

It may seem that introducing negative predication presents a solution to the problem of relevance of properties. Let us call a property *P determinate* with respect to the object *a* iff it is true that $P^*a \vee Pa$. Now one may suggest that a property is relevant to an object iff it is determinate with respect to it. Unfortunately, this suggestion is wrong. Indeed, it works in some cases, as e.g. for (1) and other similar cases when we deal with *precise* properties that are strictly defined. Statements like (2), on the contrary, cannot be explicated in this way, because the property "honest" (as well as, e.g., "wise") is extremely *vague*. According to our intuition, these properties are relevant to all people, although for many persons it is entirely unclear whether they are honest or not ($P^*a \vee Pa$ does not hold).

Nevertheless, negative predication can be very useful for introducing one crucial notion, the notion of *generalization of properties* which, in its turn, is of great importance for understanding of what the relevance of properties consists in. This notion cannot be introduced by means of a definition like: property *Q* is a generalization of the property *P* iff all objects that have *P*, have also *Q*. Such a definition is incorrect in view of so-called *coextensive properties*. For example, a couple of properties like "to be sold" -"to be bought" satisfies the definition, but these properties are by no means generalizations of each other.

Let $P < Q$ be a symbolic representation of an expression "property *Q* is a generalization of the property *P*". Then we have the following definition:

Definition 3. $P < Q \Leftrightarrow \forall x\,(Px \vee P^*x \rightarrow Qx)$.

It is easy to see that < is transitive, i.e. if $P < Q$ and $Q < R$, so $P < R$. Definition 3 shows, what is the difference between generalization of properties and the relation of simple meaning inclusion between the corresponding predicate terms: to be a proper generalization of P, Q should include not only P itself, but also its "negative counterpart". For example, there is a meaning inclusion between the terms "quadrate" and "rhomb", but a *real* generalization of "quadrate" is, of course, not "rhomb" but "geometrical figure".

In many cases we are interested not in any generalization of the given property, but in what can be called its *natural generalization*. Natural generalization of a property should represent the *natural kind* for this property. However, an attempt to *define* such natural generalization meets serious "metaphysical" difficulties.

Suppose that for every property there is exactly one other property "objectively" being its natural generalization. Let $P <_n Q$ stands for "Q is the natural generalization of P". Then a possible way of defining this relation is to find the *nearest* generalization of P. That is, we may wish to consider a definition like

(NG) $P <_n Q \Leftrightarrow P < Q$ and $\forall R\,(P < R$ and $R < Q \rightarrow \forall x\,(Rx \leftrightarrow Qx))$.

This definition is quite questionable. In fact, it easy to see that in this case Q is always $[x.Px \vee P^*x]$. This is, of course, a very unwelcome consequence of the proposed definition. Now, if we insist that we should consider only simple properties, it would be only a partial and therefore unsatisfactory solution, because generally we are not going to have only such a limited language which considerably restricts our expressive possibilities. Moreover, this definition presupposes some strong ontological assumptions, such as: the structure of the world is somehow defined in advance, and for every property *there is* the nearest generalization of it. This assumption is very doubtful indeed.

Therefore, we should give up any attempt to *define* the notion of natural generalization. This does not mean, however, that the notion itself is useless. On the contrary, in many cases it is very useful to distinguish among all the generalizations of the given property that one property which can be considered as its natural generalization. In what follows we will do it this way: having a property P, we take for granted that we can always label one of its generalizations as a "natural generalization" of P. We will mark such a property as P^{nat}.

6. Context-free relevance of properties

Consider the property "bald". To which objects is it relevant? First of all, to people -both bald and not bald. Unfortunately, this property is vague, so that many people take an "intermediate position", and it is difficult to say, whether they are bald or not. However, everybody will agree that the property under consideration is relevant to these "intermediate" people (perhaps, even more relevant than to other, "non-intermediate" people). And to which objects is it not relevant? Evidently, for chairs, tables, numbers, and all the objects that cannot be bald. And what about robots, or animals? Apparently, it would be not unnatural to admit that this property is relevant to them. (E.g., sometimes we say that lionesses are bald, as opposed to the lions.) That is, we can meaningfully consider a question whether this or that robot, or animal is bald or not. Why? Because all these objects have a head, and as such they may be bald! The property "bald" is relevant to these objects *with respect to the property* "to have a head". Let $Rel_Q(\pi P,a)$ stands for "property P is relevant to the object a relative to the property Q". Then, we arrive at the following definition:

Definition 4. $Rel_Q(\pi P,a) \Leftrightarrow P < Q \ \& \ \forall x\ (x=a \rightarrow Qx).$

The next step is to use the convention from the previous section. Suppose we mark out some property P^{nat} such that $P < P^{nat}$ as a natural generalization of P. It gives us a possibility to define the simple (context-free) relevance of properties to objects ($Rel(\pi P,a)$):

Definition 5. $Rel(\pi P,a) \Leftrightarrow \forall x\,(x=a \rightarrow P^{nat}x)$.

In words: a property is relevant to an object *a* iff, the object instantiates the natural generalization of the property (and this should hold for any object that could be on the place of *a*).

The above example with the property "bald" perfectly illustrates the definition. We may consider the property "to have a head" as a natural generalization of the property "bald". Thus, the property "bald" is relevant to any object that has a head. As another example take the property "green". Hegel tried to ascribe this property to the spirit, and to show on this way an inadequacy of formal logic. However, it can be shown that the property "green" is irrelevant to such object as spirit, and as such cannot be ascribed to it. Indeed, the natural generalization for "green" is "colored" (in the sense "to have a color"), and spirits have no color at all.

Definition 4 correlates to Dunn's definition of relevant predication, but it has an essential distinctive feature. It gives a possibility to explicate such cases as (1) and (2), whereas Dunn's definition does not. (In fact, Dunn does not aim at the analysis of this sort of predication, his goal is more narrow -to rule out predications like (4).)

This definition gives us the context-free relevance, because we do not take into consideration any additional context. Below we will consider the role of context and introduce the notion of relevance that allows to analyze the contexts where the property of "being such an *x* that Socrates is wise" can be relevant to Alcibiades.

7. Complex predicates: a general case

Before we come over to context-dependent relevance of properties, we have to extend the definitions above on complex predicates. The main problem consists in the negative predication -what should be the scheme of abstraction in this case? We have to take into account that the "star"-operator can be applied only to simple predicates. Thus, we need special rules for "importation" the star into complex predicates.

Definition 6. If X is any formula of the language, then $(X)'$ is the *negative reflection* of X which is constructed in accordance with the following conditions:

(a) $(Px)' = P^*x;$
(b) $(P^*x)' = Px;$
(c) $(A \& B)' = (A)' \vee (B)';$
(d) $(A \vee B)' = (A)' \& (B)';$
(e) $(\sim A)' = \sim(A)';$
(f) $(\forall xA)' = \exists x(A)';$
(g) $(\exists xA)' = \forall x(A)'.$

It is important that $(X)'$ itself is not a formula of the language, but an abbreviation of the corresponding formula that is built up by (a)-(f).

Now we can introduce the abstraction scheme for complex predicates:

(NA2) $[x.Ax]^*a = (Aa)'.$

(NA2) makes it possible to generalize the definitions for context-free relevance of properties:

Definition 7. $[x.Ax] < [x.Bx] \Leftrightarrow \forall y\, ([x.Ax]\, y \vee [x.Ax]^*y \rightarrow [x.Bx]y).$

Definition 8. $Rel_{[x.Bx]}(\pi[x.Ax],a) \Leftrightarrow [x.Ax] < [x.Bx]\ \&$
$\& \forall y\, (y=a \rightarrow [x.Bx]\, y).$

Definition 9. $Rel(\pi[x.Ax],a) \Leftrightarrow \forall y\, (y=a \rightarrow [x.Ax]^{nat}y).$

8. Context-dependent relevance of properties

Now we are approaching a very interesting topic: how a context can affect the relevance of properties. The main goal of Dunn's analysis is to rule out cases like (4) which *all*, according to Dunn, represent irrelevant predication. Using definition 1, he explicates (4) as "Irrelevant Predication" in the following way:

(IP) $(\rho xP)a \Leftrightarrow \forall x\ (x{=}a \rightarrow P)$.

Dunn remarks that the right-hand side of this expression corresponds to a failed relevant implication (even when *P* is true), and concludes:

> An *x* being identical to Alcibiades has nothing to do with Socrates being wise (Anderson, Belnap and Dunn 1992, p. 455).

However, we are going to show that in *some contexts* this may be the case. Suppose, for a wile, Socrates be a pupil of Alcibiades. Then a sentence like "Alcibiades is such that Socrates is wise" appears to be not as strange, as on the face of it. Another example of this kind:

(9) The iceberg is (was) such that the ship went down in 15 minutes.

Propositions (4) and (9) have essentially the same structure. It is clear however that the property of being such (an *x*) that a ship went down in 15 minutes *can be* relevant to an iceberg. But only to the iceberg that has collided with the ship! Thus, one may notice that in both cases there is some additional *relation* between the objects before and after the "such that"-expression. Just this relation makes the whole "such that"-construction relevant. We state that this relation can be represented by the so-called *as-(*or *qua-)* constructions. Indeed, suppose that Alcibiades *is such that all his pupils are wise* (i.e. we consider Alcibiades as such an *x*, that all the pupils of *x* are wise), and that Socrates is a pupil of Alcibiades.

Let us remember that Dunn explains the case with Alcibiades and the property of being such an *x* that Socrates is wise using (4'). He notes that (4') has the form of (PP), and states that "there is no way in the world with 'no funny business' that one can get A, *using* B". We are going to show, however, that under the above mentioned assumptions such a way can be found. Moreover we can stay exclusively within the conceptual framework proposed by Dunn. Indeed, we have:

(10) Alcibiades is such that all his pupils are wise. Socrates is a pupil of Alcibiades. Thus, Alcibiades is such that Socrates is wise.

To make the point evident we can transform (10) into the following (Dunn-style) inference:

(10') Alcibiades is such that all his pupils are wise. Therefore, if anyone is Alcibiades then all his pupils are wise. Socrates is a pupil of Alcibiades. That is, if anyone is Alcibiades then Socrates is wise.

Note, that we do not need the premise "Socrates is wise" to get to the conclusion in (10'), but *do* need the premise "if anyone is Alcibiades..."! In this case the fact that anyone is Alcibiades is relevant to the fact that Socrates is wise, and hence the property of "being such that Socrates is wise" is relevant to Alcibiades himself.

It is interesting that the analysis of (10') is made exclusively in spirit of Dunn's theory. We only involve some additional assumption on the relations between Alcibiades and Socrates. Nevertheless, definition 1 does not allow to reflect this additional context.

The role of a context can be explicated in different ways. For the purposes of the present paper it is appropriate to reflect this role by introducing "*as-expressions*" (cf. the analysis of "*qua*-being" in Poli 1994). In accordance to such understanding, we get the notion "the property *F* is relevant to *a* as being some *x*".

Let $a\backslash x.Ax$ means "*a* as such *x* that *Ax*". What does it mean that a property is relevant to an object *as some x such that Ax*? It means that aside from the information that an object should be equal to *a*, we have to take into account the additional data that the object is such that *Ax*. This motivates the following definition:

Definition 10. $Rel(\pi P, a\backslash x.Ax) \Leftrightarrow \forall y\ (y{=}a\ \&\ Ay \rightarrow P^{nat}y).$

For complex predicates:

Definition 11. $Rel(\pi[x.Ax],a\backslash[y.By]) \Leftrightarrow \forall z\ (z{=}a\ \&\ [y.By]z \rightarrow [x.Ax]^{nat}z)$.

Consider the iceberg as such that it has collided with a ship (see (9)). The property of being such an *x* that the ship went down can be considered as the natural generalization of the property of being such an *x* that the ship went down in 15 minutes. Then the property of being such an *x* that the ship went down in 15 minutes is relevant to an iceberg iff for every object if it is the iceberg and has collided with the ship, the ship went down. An analogous analysis allows to explicate the above context where the property of being such that Socrates is wise is relevant to Alcibiades.

In the present paper we do not consider the case when a context can turn a relevant property into an irrelevant one. However, a context can influence the relevance of properties also in this way. For example, in Shramko 1994 one can find some contexts in which "sweetness" is not a relevant property for a rose.

9. Some facts

Concluding the paper we establish some formal results that reflect important features of the introduced notion of relevance of properties. We will deal with simple properties, having in mind that all the proofs can be easily extended to complex predicates.

First we clarify the relations between Dunn's notion of relevant predication (which we call the non-fictional predication) and our notion of relevance of properties.

Fact 1. If an object non-fictionally (in the sense of definition 1) has a property, than the property is relevant to the object.

Proof.

1. $\forall x\ (x{=}a \rightarrow Px)$ (definition 1)
2. $\forall x\ (Px \vee P'x \rightarrow P^{nat}x)$ (definition 3)
3. $Px \rightarrow P^{nat}x$ (2, $\forall$-elimination, propositional logic)
4. $\forall x\ (x{=}a \rightarrow P^{nat}x)$ (1, 3, $\forall$-el., transitivity, $\forall$-int.)

Let us say that the property is *non-fictionally determinate* with respect to *a* iff $\forall x\ (x{=}a \rightarrow Px \vee P'x)$. Then we can generalize the fact 1 in the following way:

Fact 2. If a property is non-fictionally determinate with respect to *a*, then it is relevant to it.

Proof.

1. $\forall x\ (x{=}a \rightarrow Px \vee P'x)$ (*P* is non-fictionally determinate wrt *a*)
2. $\forall x\ (Px \vee P'x \rightarrow P^{nat}x)$ (definition 3)
3. $\forall x\ (x{=}a \rightarrow P^{nat}x)$ (1, 2, $\forall$-el., transitivity, $\forall$-int.)

Finally we show that the property of being self-identical cannot create a context that may influence relevance of any other property.

Fact 3. If a property is relevant to an object as being such that it is self-identical, then the property is simply relevant to an object.

Proof.

1. $\forall x\ (x{=}a\ \&\ x{=}x \rightarrow P^{nat}x)$ (*P* is relevant to *a* as such an *x* that *x*=*x*)
2. $x{=}x$ (reflexivity of identity)
3. $\forall x\ (x{=}a \rightarrow P^{nat}x)$ (1, 2, $\forall$-el., propositional logic, $\forall$-int.)

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