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THINGS, FACTS AND EVENTS

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Uwe Scheffler and Yaroslav Shramko

THE LOGICAL ONTOLOGY OF NEGATIVE FACTS

On what is not

"... we see that the lemon is yellow,
we do not see that it is not blue."
(A. Grzegorczyk)

Abstract. We consider Russell's theory of negative facts and construct a suitable theoretical model that provides an ontological basis for this theory. Proceeding from the hypothesis that negative facts cannot be eliminated and cannot be reduced to any sort of positive facts, we argue that negative (atomic) facts should be logically expressed by means of a special type of predication – a negative predication. The introduction of negative properties enables us to justify the essential discrimination between positive and negative predications. In the final section, we present a model-theoretical semantics for negative facts.

1 Introduction

According to a theory of facts as "truth-makers", a

(1) ... sentence A is true {false} if and only if some fact that makes A true {false} is the case. (Fraassen 1975, p. 222)

Thus, for every true sentence there is a corresponding fact that makes it true. As this holds for all sentences, (1) together with the structure of the applied language leads to a classification of facts into elementary (atomic)

facts on the one hand, and compound ones on the other. First of all, this concerns conjunctive propositions: theories of the kind mentioned above usually imply the acceptance of "conjunctive facts":

(2) For every two facts e and e' there is a conjunctive fact $e \cdot e'$ that is the case if and only if both e and e' obtain. (Fraassen 1975, p. 223)

Conjunctive facts can be interpreted in terms of co-existence, they do not cause formal or intuitive difficulties. Serious problems arise if one tries to use a similar approach to disjunctive and negative propositions. As to negated sentences, the question is: what kind of facts makes them true? Suppose a negative proposition non-A, A being an elementary sentence. Consider the following three possible answers to the question above:

(3) There is a *positive* atomic fact that makes A false (i.e., that makes true some elementary positive sentence B which is incompatible with A);

(4) there is a *compound* negative fact that makes non-A true;

(5) there is an *atomic* negative fact that makes non-A true.

R. Demos in (Demos 1917) defended (3), which certainly is the most challenging idea to all theories recognizing negative facts. There, he denied the existence of negative facts at all.

> Demos starts his discussion with an observation: simple singular negative statements exist and they do not depend on a special attitude of utterers to them. This seems to imply the existence of corresponding negative facts, but, as Demos emphasizes, such facts are not met within experience. He grounds this remark – which is essential for his argument and is by no means obvious to us – on an interrogation among some persons. The possibility of using negative predicates (prescribing "not-white" instead of denying "white") he rules out by the observation that one may deny $P(a)$ not only because a is not-P, but also because a does not exist.
>
> Demos introduces a relation of opposition between (positive) facts, and then interprets the negation of a sentence as a claim about an opposite of a positive fact: "not-A" is really "some proposition B is true which is a contrary (an opposite) of A". According to him, a negation – when applied to a simple singular sentence – is a description-like operator which refers to an entity (here: to a positive fact) without mentioning it. Therefore, negative propositions refer to positive propositions, and these in their turn assert positive facts (cf. Demos 1917).

Although even some of Demos' own examples invite quite strong arguments against Demos' approach, we will not discuss his position here. Instead, we will refer to an authority: B. Russell criticized this doctrine and insisted that negative facts occur.

> They are irreducible at least in two senses. First, the incompatibility relation "opposition" constitutes negative facts, such as the one that "John's being at home" and "John's being at the shop" are opposite to each other. The corresponding sentences cannot be true together – even if we take that rather to be an intuitive explanation than to be a definition, it involves negation. Hence, there are negative facts. Second, a complete description of the world should mention not only how it is, but also how it is not (Russell 1992, pp. 211–215).

It is not entirely clear, whether Russell considered negative facts to be atomic or compound. Some of his remarks in (Russell 1992) suggest that he shared a position close to (5), but he never explicitly claimed that negative facts are atomic. He claimed that they are irreducible. If Russell's argument concerning the irreducibility of negative facts is correct, any philosophy of facts must deal with some kind of negative facts.

It is not uncommon to accept negative fact as parts of an ontology. Usually, though, one would prefer to handle them as complex, structured, compound entities. This idea seems to be expressed paradigmatically in Grzegorczyk's famous statement:

> "The compound sentences are not a product of experiment, they arise from reasoning. This concerns also negations: we see that the lemon is yellow, we do not see that it is not blue." (Grzegorczyk 1964, p. 596)

A part of this quotation is the epigraph to the present paper. Contrary to its usual purpose, the epigraph does not express the main idea of the paper in a concentrated form. Instead, we use the quotation as a short and almost aphoristic expression of the idea which is disputed here. Although it will be shown below that some kernel of Grzegorczyk's argument remains undoubted after a slight specification, the epigraph does not indicate the position we are going to defend, but rather that one we want to contest.

Probably, and despite the results of Demos' interrogation, the most serious argument in favor of the existence of negative facts comes from human experience and practice: the lemon *is* not blue, and we do *see* this fact. It is much easier to presuppose a corresponding negative fact, than to postulate a positive fact (that it is yellow) and an incompatibility relation of facts. Indeed, sometimes one would have to *postulate* such a corresponding positive fact because it is not given to the senses. So, we may not know what color the lemon is but nevertheless see that it is not blue. It might be of a color which we are unable to determine or even to name. Furthermore, it is perfectly correct to say "I did (could) not see what color that thing was, but I definitely saw that it was not blue". In both cases the supposed incompatible positive fact making "It is not

blue" true is an unspecified fact expressed by "It is of some other-than-blue color", which is at least as suspicious as negative facts. Consider another example: A large part of any police investigation consists of carefully collecting (atomic) *negative* facts. This takes place, e.g., during the drawing of a phantom picture based on the evidence of eyewitnesses. Often, people are not sure about all the positive details, but know exactly about negative characteristics of the suspect: not tall, no mustache, eyes not blue, and so on. Their statements express nothing but atomic negative facts which sometimes are important enough to let the detective succeed. Due to such examples we take for granted that *there are atomic negative facts.*

Once accepted, negative atomic facts give raise to two principal questions. The first question consists in asking how such facts should be expressed in a language. Propositions of the form $\sim A$ are compound, hence we may ask whether they correspond to facts of the required kind. Possibly, we should use some new language features. This is the problem of an "epistemic" justification of negative facts. The second and probably more important problem concerns the ontological analysis of negative facts. Starting from logical atomism, how should negative facts be built in into the structure of the world? Are they derived or basic? We may refer to this complex as the ontological justification of negative facts. In this paper we are dealing with both problems, but we mainly pay attention to the latter one. We believe that if we succeed in finding an appropriate answer to both the problems we stated, we also obtain the answer to a general philosophical question:

How are atomic negative facts possible?

2 Facts and propositions. Facts as truth-supporters

Theories of facts as "truth-makers" based on (1) have two undesirable consequences:

(6) The occurrence of a fact is a necessary condition for the truth of every sentence.

(7) The occurrence of a fact is a sufficient condition for the truth of every sentence.

From a certain point of view, both these consequences are highly doubtful. On the one hand, many propositions are not made true by facts. Consider, for example, logically valid propositions (the laws of logic). It would be quite artificial to say that there is some fact which makes true, e.g., the law of excluded middle. On the contrary, logically true sentences are

true *independently* of what facts occur. Hence, it should be reasonable to admit true sentences which do not require the existence of particular facts. On the other hand, many sentences are not made true by facts only. The truth of such sentences includes some reasoning as a necessary component (cf. the quotation of Grzegorczyk above): to become true, these sentences have to be proved. Grounded on these intuitions we arrived at a crossroads for further explication. One may accept the existence of compound facts of all kinds as part of an ontology granting equal rights to the atomic ones. Such an approach implies the existence of a fact for every true sentence, including *logical* facts, corresponding to logical (or analytical) truth. A traditional theory of truth-*makers*, which interprets (1) as a definition of the truth of sentences, then seems to require compound logical facts *prior* to sentences. This has been done many times. Another variant consists in accepting the criticism concerning claims (6) and (7): One may try to construct an ontology which includes fewer, but intuitively well founded facts, which participate in different ways in making sentences true. This idea is in accordance with the so called *new dogma of logical empiricism*, formulated by Wessel as follows: "Observation sentences are of the form $\neg P(s^1, \ldots, s^n)$ or $P(s^1, \ldots, s^n)$ only, where $n \geq 1$ and P is a n-place predicate" (cf. Wessel 1994, p. 377).

Here, we take the second branch and propose some weaker theory about the role of facts in the truth of sentences. Its very essence consists in distinguishing between two types of "making true", a direct and an indirect one. Besides the direct correspondence between facts and some sentences – which is called for by (6) and (7) –, there is an indirect one, according to which a fact merely *participates* in making the sentence true. Let us keep the term "makes true ..." for what we described as "makes true directly", and let us introduce a new term "supports the truth of..." for what we described as "makes true indirectly". For many sentences, we should say that facts *support* the truth of these sentences, rather than claiming that these are *made* true by the facts. Thus, we propose a theory of facts as "truth-supporters". The main principles of such a theory are as follows:

(8) An elementary sentence A is true if and only if some fact makes A true is the case.

(9) A fact supports the truth of a sentence A if and only if this fact makes true some sentence from which (maybe also using some other true sentences) A may be deduced.

Obviously, truth-making is preserved for elementary sentences, while the others can only be supported by some facts. This approach has some

interesting consequences:

(10) We do not need compound facts. All the facts are atomic. From now on, facts *are* atomic facts.

(11) Grzegorczyk's view that "the truth of compound sentences arises from reasoning" is correct, but has to be completed by – "and from facts, both together".

Indeed, the sentence $p \wedge q$, for example, becomes true as a result of an application of the rule

$$p, q \Longrightarrow p \wedge q$$

and the truth of this sentence is supported by the facts that **make true** the sentences p and q.

(12) A sentence may be supported by several different facts.

(13) A fact may support several different sentences.

(14) As we have (9), for a logic of "truth-support" we need some theory of deduction, for which the rule "weakening" and the "fallacies of relevance" do not hold. E.g., a suitable system of substructural logic may be of use here.

Notice, while (1) deals with truth and falsity simultaneously, "making false" or "supports falsehood" were not mentioned yet. Usually, facts are constructed in order to make sentences true, but also in order to back up the falsehood of all other remaining sentences. As Russell put it:

> When I speak of a fact ... I mean the kind of things that makes a proposition true or false. (Russell 1992, p. 182)

Such a position essentially presupposes a "duality of truth and falsehood" (Russell 1992, p. 187). That is, "truth" and "falsehood" both are taken as primary independent values for propositions. To preserve the principle of bivalence (which is crucial for classical logic), one has to accept some additional conditions that regulate the inter-relations between these values (these are namely the conditions of completeness and consistency – every proposition should be either true or false, and propositions cannot be true and false simultaneously). We believe that this approach actually implies a *four-valued* theory of truth-values. Indeed, such a theory can easily be developed based on the above mentioned duality. All that is needed is to "play around" with the conditions and to change therein the relations between "truth" and "false". This has been done: for example, one can mention the well-known "semantics of American plan" of relevant logic

(see, e.g. Dunn 1976, Belnap 1977), which side by side with T (true) and F (false) accepts the values *None* (neither true, nor false) and *Both* (true and false).

We believe that a purely classical approach to logic presupposes a "monism" of truth being *the only* initial value. "False" is nothing but another name of "not true", or (which is the same in the classical case) is equivalent to "true that not ...". The real bivalence is between "truth" and "non-truth". In this sense, "falsehood" is a redundant notion. Stating that a sentence A is false, one actually indicates "A is not true" ("it is true that $\sim A$"). There is always some reasoning involved in getting to such a statement: one has to start with elaborating the question whether or not there are facts supporting the *truth* of A. Therefore, it is not legitimate to say that some fact supports the falsehood of a sentence – facts may support truth only.

Consider the example concerning the police investigation again. The job done by a detective may serve as an acceptable illustration of the theory of facts as truth-supporters. Facts play a very important role in a detective's business – to disclose the crime, he has to be very diligent in collecting facts. Nevertheless, an investigation seldom does not go beyond such a "fact-collecting stage". In most cases a detective comes to the truth through some process of reasoning, although the reasoning always has to rest upon the facts. This "deduction stage" is the most interesting and intriguing part of investigations, and this stage usually draws the main attention of writers of crime stories and their readers.

This "deduction stage" is involved in any kind of evaluation of compound sentences. Sometimes many steps have to be made, sometimes only one. For negative sentences such a view implies that we shall agree with Grzegorczyk. True negative sentences which are compound propositions (i.e., sentences with an ordinary truth-functional negation as the main connective) are always results of reasoning (although, they may well be supported by facts). Such sentences, obviously, cannot express genuine atomic negative facts. Which sentences are made true by atomic negative facts according to (8)? In section 4 we will introduce and consider such sentences, thereby answering the question how negative facts can and do find their expression in a formal language. As one should expect, there is a relationship between atomic negative facts, atomic negative sentences, and certain negative sentences as compound propositions.

Generally speaking, it is not sufficient to have knowledge about all facts in order to evaluate all sentences. Many of them involve language rules and logic. It is our aim to track down this undoubted epistemic judgment to an ontological level: (atomic) facts, by the help of logic, are the only truth-makers. In a sense, facts and logic constitute the world.

So, first we need a theory of facts based on what there is, then we will go ahead to what is not.

3 The structure of facts. Things, properties and relations

In the philosophy of logical atomism facts are considered as some "logical atoms" (Russell 1992, p. 179). As Wittgenstein put it:

> The world is determined by the facts, and by these being *all* the facts. (Wittgenstein 1961, 1.11)

It was he, and – following him – Russell, who claimed that facts, rather then objects, are metaphysically basic. This does not mean, however, that facts do not have an internal structure, or that this structure cannot be analyzed. It does not even mean that facts are independent of all other entities. The situation is similar to the case of physical atoms – for many purposes it is enough to consider them as "solid" units, but in many other cases their structure is of great importance. As we are interested in negative facts, we have to look into their internal structure.

Facts are not independent entities. We find facts by considering objects and their properties and relations. Facts belong to the realm of how people organize their comprehension of the world, of how they describe their experiences. As Wittgenstein pointed out, it is the objects that form the "substance of the world" (see Wittgenstein 1961, 2.021). That is, objects (or things) have the most real "being". Objects have properties and are in various relations to each other. Properly speaking, there is – in a first step – nothing else in the world but things that have properties and are in relations to each other. Properties do not have an independent being – they can exist as properties *of objects* only. An object is given together with all its properties. Although there is no object without any properties, the ontological status of objects and properties is not the same: just objects – and objects only – are bearers of properties and relations.

Certain objects have certain properties, others do not have them. This meta-relation between objects and properties is sometimes called *instantiation*. Its "ontological status" is intricate, because it takes properties as an argument. One may interpret it as expressing how objects have properties, thereby making facts possible. Every fact is an instantiation of a property by an object (of a relation by an n-tuple of objects), every atomic sentence expresses such an instantiation. Let A be an atomic sentence, then we may refer to the corresponding fact by $\downarrow A$. The term $\downarrow A$ is a name of a fact, as far as A does not contain any fact terms, $\downarrow A$ is a primary fact. (More formally, one should regard $\downarrow$ to be a term-forming operator generating subject terms from sentences. For details cf. Wessel 1995.)

Since facts may have properties and stand in relations between each other and to objects, it is possible to form true sentences about facts. Russell, for example, spoke about facts and objects using the relation of "being a component", we used properties of facts like "atomic" or "occurring". The corresponding facts – which should exist, once we decided to allow for fact terms – are facts about facts and are called *secondary* ones. We suppose here that some of the secondary facts play a special role: they are laws of nature. Law statements claiming, e.g., certain relations between pressure, temperature and the volume of a gas are about certain types of facts, the law itself is a secondary fact. At the moment, we are not interested in clarifying the metaphysical and ontological status of secondary facts. There are two main reasons why we mentioned them at all: First of all, we want to emphasize that if one decides to consider facts, they belong to the ontology by equal rights. Nevertheless, they are dependent in the Strawsonian sense: it is impossible to identify facts without identifying objects, but it is possible to identify the latter without identifying the former. Moreover, facts are language dependent, which is quite obvious in case of the secondary facts. The second reason for our mentioning these facts is a puzzling problem: How should we deal with "real" hypothetical sentences? We want to hint at a solution, without discussing the details:

> Universally quantified conditional sentences which are laws of logic, language rules (meaning postulates, consequences of definitions, and so on), and laws of nature do not provide any difficulties. Their truth either is not supported by facts, as was mentioned above, or is directly made true by secondary facts, and therefore supported by them. Universally quantified conditional sentences about a finite universe can be dealt with as if they were (quite long, possibly) conjunctions. Singular conditional sentences ("If that particular object was a P, it would be a Q") are supported by corresponding laws of logic, language rules, secondary facts, and, possibly, by other facts. The only remaining problem is that of universal sentences true by chance which are not laws, not language rules, but hold in infinite universes. However, it is still an unsettled question whether or not such sentences exist at all. This problem goes far beyond the subject of the present paper and involves very sophisticated philosophical considerations.

Facts of all kinds, thus, are part of our ontology. At the moment, we are discussing primary facts only, leaving secondary facts to further investigation.

4 Two types of predication

The structure of the world described above finds its precise reflection in the logical structure of our language. Objects, properties and relations

are denoted by special terms. We have singular terms $(a, b, c, \ldots)$ for objects, sufficiently many one-placed predicate terms $(P^1, Q^1, R^1, \ldots)$ for properties, and two, three, etc.-placed predicate terms for binary, ternary, etc. relations. There is also a *metaoperation of predication (Pred)*, by which predicate terms are ascribed to singular terms. Thus, when we wish to ascribe some predicate term P^n to the singular terms $a_1, \ldots, a_n$, we just write – $\mathrm{Pred}[P^n, a_1, \ldots, a_n]$. The expression $\mathrm{Pred}[P^n, a_1, \ldots, a_n]$ does not belong to the object language. It is a metastatement about the objects $a_1, \ldots, a_n$ instantiating the property (relation) P^n. Ascribing an one-placed predicate term to a singular term expresses the fact that the corresponding object has the corresponding property, and ascribing an n-placed predicate terms to n singular terms – that there is a relation between the objects. The following *scheme of predication* shows how we obtain elementary sentences that express such facts:

(15) $\mathrm{Pred}[P^n, a_1, \ldots, a_n] \Longleftrightarrow P^n(a_1, \ldots, a_n)$.

Thus, we strongly distinguish between metastatements of instantiation like "Socrates instantiates the property of being wise" and the corresponding object language sentences as "Socrates is wise".

Some authors have suggested considering a *negative predication* as a special type of the predication operation besides positive predication. Such a negative predication cannot be equivalent to the truth-functional negation of the positive case of predication because it produces atomic sentences. The idea is that the statement that it is not the case that an object has a property is not equivalent to the statement that an object does not have this property. The latter is stronger than the former. As R. Turner, developing "a theory of properties", mentioned:

> ...the 'internal negation' of properties is not always definable in terms of sentential negation. ... We can institutionalize this by introducing 'negative' predication relation (Turner 1987, p. 459)

F. Kamareddine in (Kamareddine 1992) also considers the possibility of distinguishing between positive and negative predication.

In the 60–70ties, A. Zinovjev and H. Wessel carefully elaborated a theory which deals with both types of predication (see, e.g., Sinowjew and Wessel 1975). They call their theory a "non-traditional theory of predication" (NTP). The backbone of NTP consists in assuming negative predication as well as positive predication and distinguishing them syntactically. Let $\mathrm{Pred}^{\ominus}$ stand for negative predication. Then, $\mathrm{Pred}[P^n, a_1, \ldots, a_n]$ and $\mathrm{Pred}^{\ominus}[P^n, a_1, \ldots, a_n]$ both represent elementary facts, being *almost* independent of each other. The word "almost" occurs, because one restriction still exists: the corresponding facts cannot be simultaneously the case (although they both very well may fail to be the case). The only postulate we need in order to express the relationship between Pred and $\mathrm{Pred}^{\ominus}$ is the following:

(16) $\mathrm{Pred}[P^n, a_1, \ldots, a_n]$ and $\mathrm{Pred}^{\ominus}[P^n, a_1, \ldots, a_n]$ cannot be the case simultaneously.

The analogue of the *Law of Excluded Middle* does not hold for Pred and $\mathrm{Pred}^{\ominus}$. Below, we will need another important feature of (NTP):

(17) If $\mathrm{Pred}^{\ominus}[P^n, a_1, \ldots, a_n]$ is the case, then $\mathrm{Pred}[P^n, a_1, \ldots, a_n]$ is not the case (but not *vice versa*).

Syntactically, $\mathrm{Pred}^{\ominus}$ find its expression by means of the so called "internal negation" ($\neg$) through the following predication scheme for negative predication:

(18) $\mathrm{Pred}^{\ominus}[P^n, a_1, \ldots, a_n] \Longleftrightarrow \neg P^n(a_1, \ldots, a_n)$.

Clearly, internal negation is not a usual truth-functional negation, it serves merely to express the negative type of predication. By (18), its iteration is impossible, and it can appear in front of elementary sentences only (more correctly: only as a part of elementary sentences). Now we have the following object language counterparts of (16) and (17):

(19) $\sim(P^n(a_1, \ldots, a_n) \wedge \neg P^n(a_1, \ldots, a_n))$.

(20) $\neg P^n(a_1, \ldots, a_n) \longrightarrow \sim P^n(a_1, \ldots, a_n)$.

With the help of (19) and (20) based on NTP we got a possibility to differentiate between the absence of a positive fact and the occurrence of a parallel negative fact. Loosely speaking, whereas $\sim P(a)$ represents sentences like "It is not true (not the case) that the lemon is blue", $\neg P(a)$ expresses the statement "The lemon is not blue". These sentences are sometimes used as if they were synonymous, but they are not synonymous in all circumstances. We use the first sentence if some positive fact is not the case; we use the second sentence if some negative fact is the case. The following example shows the difference:

> It is not the case that the refrigerator is honest, hence the sentence "The refrigerator is honest" is false and its truth-functional negation is true. But it is also not the case that the refrigerator is dishonest, hence the sentence "the refrigerator is dis-honest" (with "dis-" acceptably interpreted as internal negation) is false, too. The property's being not instantiated by the refrigerator does not imply the occurrence of the parallel *atomic negative* fact, although, as we will see in a moment, the converse may appear.

Thus, we state in accordance with NPT:

(21) In our language, negative (atomic) facts have to be expressed by means of elementary sentences with an internal negation. There is a special type of predication relation – the negative one (a kind of denial or rejection), which represents negative facts in the language.

Now return to Grzegorczyk's statement that compound sentences arise from reasoning. What kind of reasoning do negative sentences with truth-functional negation arise from? Or, involving the "factual" level, what kind of facts support the truth of such sentences? Consider our (or Grzegorczyk's) lemon once again. One may suppose that the truth of the sentence

(22) It is not the case that the lemon is blue

is guaranteed by the following short argument:

(23) The lemon is yellow. Nothing can be yellow (everywhere) and blue (everywhere) simultaneously. Hence the lemon is not (in the sense of "it is not the case") blue.

Thus, one fact that supports the truth of (22) would be the fact that the lemon is yellow (and Grzegorczyk states just this). But what about the statement that nothing can be yellow and blue simultaneously? We need this statement to obtain (22), the mere observation of the "yellowness" of the lemon is not sufficient. This special universal sentence can never be fully reduced to primary facts. It is true due to our usage of basic color terms: we use them as excluding each other. Along this line of reasoning, Grzegorczyk's remark finds an interpretation in a, so to speak, Demos-compatible sense. But there is another line, too:

(24) The lemon is not blue (I can see this and I do believe my eyes). Hence, it is not true that the lemon is blue.

The argument in (24) depends on the existence of a negative fact which makes true the premise sentence. Making it true, this fact supports by (17) or (20) the truth of (22).

5 Negative properties. The operator of contrariety

In the section above, we were dealing with the *epistemic* possibility of negative facts. It is based on a special negative type of predication that is present in our language. The question whether and how negative facts are *ontologically possible* is still open. A world which is described as in section 3 does not contain anything equivalent to a negative predication relation. Instantiation has no negative counterpart, at least it is hard to

imagine a meta-relation relating things to properties which they *do not* have. What, then, is the ontological background of negative predication? We will try to answer this question by distinguishing between two types of properties.

Properties are treated in various "property theories". Most of them meet a general requirement, namely that the class of properties should be closed with respect to logical connectives and quantifiers (see e.g. Chierchia and Turner 1988, p. 262). This requirement finds its reflection in the principle of β-conversion:

(25) $[\lambda x\, A(x)](a) \Longleftrightarrow A(a)$,

where the expression $\lambda x\, A(x)$ is read as "the property of being (an x such that x is) A". As A is unspecified, we can speak about "conjunctive", "disjunctive" ... properties like "the property of being clever and tall", or "the property of being yellow or blue".

The apparatus of λ-calculus enjoys both technical expressiveness and philosophical finesse. We would like to state, however, that constructing predicates by means of λ-abstraction reflects rather the human capacity to abstract than really existing properties. We believe that real properties are all simple, like "red", "tall", ..., and that "disjunctive", "conjunctive" or "implicative" predicates, created by the corresponding λ-expressions, are pure results of abstraction and do exist only in our language. There is no one-to-one correspondence between the language and the world, the former is more complicated (in certain respects), and these complications are brought in by human activities. We need expressions for complex properties for many purposes, especially for keeping our arguments short and understandable. Nevertheless, predicates such as the well-known "grue" show some of the difficulties arising from predicate-building procedures. Of course, the notion of "real" existence itself – in contrast to pure fictional or hypothetical existence – is extremely vague with respect to properties. Does the property "bleen" *really* exist side by side with "blue" and "green"? We believe that this example shows the impossibility of an entirely language independent ontology of properties. The existence of properties depends not only on objects which instantiate them, but also on the language which makes it possible to perceive them. This is reflected by the fact that there is a special sort of terms in our language, namely predicate terms, which are meant to express properties and relations. Actually, only simple predicate terms serve as primary terms of a language. Correspondingly, simple properties constitute the base of the properties' ontology. Therefore, we may confine the consideration with those properties which are ontologically basic. Thus, we state:

(26) A predicate is called simple, if and only if the result of applying

β-conversion to the corresponding λ-expression is an elementary sentence. The properties that are expressed by simple predicates we call simple properties.

Further, one may observe that at least some predicate terms, even those which express simple properties, have a *negative form*. Among such negative predicates are, e.g., "dishonest", "infinite", "inconsistent". According to a certain point of view (cf., for example, Halbasch 1971), all negative properties can be removed from the discourse – we believe this view to be wrong. Obviously, many negative properties *cannot* be eliminated from the discourse. Of course, for many negative predicates one can offer a positive predicate that has approximately the same "sense": "dishonest" means "mendacious"; "inelegant" means "awkward"; "uncertain" means "vague", etc. Very often however, such substitutions are not perfectly equivalent. It is doubtful indeed, whether "dishonest" is precisely the same as "mendacious", and "inelegant" – the same as "awkward". The main point is that for many negative predicates there is simply *no* parallel positive predicate, and the "essence" of such predicates consists in expressing a – so to speak – "negative idea". Consider, e.g., "indescribable", or "unpaid", or "unjustified", or "impossible" (the examples can be continued). Even while trying to define these (and many other) predicates directly, we use negative terms.

Negative predicates express really existing negative properties, and the main feature of a negative property is its opposition to some positive property. One can consider a special kind of negation that corresponds to such a relation of opposition – a contrare negation. In logic, there was a long tradition of distinguishing between "contrare" and "contradictore" types of negation. As A. Fuhrmann (Fuhrmann 1997) points out, this division disappeared – aside from few exceptions – in modern symbolic logic. This was paid off by excluding contrare negation from consideration (as if there was no such sort of negation at all). The reason for this development is the ideological predominance of *propositional* logic from the very beginning of modern logic. Propositional logic knows only one type of negation – a truth functional negation which forms sentences from sentences. So there is nothing especially surprising in the fact that from a purely "propositional" point of view the above mentioned kinds of negations "merge" into one external negation. The contrare type of negation is located *inside* the simple sentences (because it is an *internal negation*, it does not negate the sentence), but within propositional logic the internal structure of sentences is not considered at all. NTP deals with the internal structure of simple sentences, its language uses a negative predication in order to express the idea of internally denying instantiation. Negative

properties may do the same job on the ontological level.

Without a doubt, many properties have negative counterparts. It is less trivial to find an answer to the question what the oppositeness of properties consists in. That is, how can we say that some property is a negative counterpart of another? The answer we suggest here is (contrary to what is claimed, for example, in (Halbasch 1971), p. 399): the contrariety of properties is expressed *only* by logical means, it is due to logical form only. Usually, people use the various "not", "dis-", "un-", ... of natural languages for these aims. White and black, male and female, dead and alive are not real oppositions, but only white and not-white, dead and not-dead etc. To state that white and black, dead and alive are oppositions, one always has to use some additional *factual* information, presuppositions etc. (e.g., a suitable theory of colors). But such presuppositions may change, and hence, if we try to treat the contrariety of properties by taking into account additional factual considerations (e.g. about the physical or chemical nature of properties), it could well happen that what is "opposite" in one relation, is not opposite in some other. Following this approach, our task should consist in making transparent the underlying logical structure.

Before we proceed in this direction, we want to meet a possible objection. In (Neale 1988), Neale considers among others the problem of perceptual reports of negative facts – as in "Adam saw Mary not wink", for example. Discussing a suggestion to explicate such natural language structures by the help of a corresponding antonymic predicate *wink'*, instead of utilizing classical negation (which would be our solution, too), he raises the problem of so called natural antonyms. Indeed, think about predicates such as "leave" and "stay", or "even" and "odd". In both pairs, the former behaves as if it is the parallel negative predicate of the latter, and *vice versa*. The question is whether the relation between natural antonyms is the same as that between positive and (parallel) negative predicates related by "not", "dis-", "un-", or is it that of "black and white". The first choice obviously leads to difficulties: Claiming that "leave" *is* or *is defined as* "not-stay" and *vice versa* contains, first, a circle in the definition, and, second, calls for iterations of "not". Both consequences are unwanted. Nevertheless, a theory dealing with negative predicates must explain why people practically use "leave" as if it is the formal contrare negation of "stay", why it is possible to infer not only

(27) Adam didn't see Mary leave

from

(28) **Adam saw Mary not leave,**

but also

(29) Adam saw Mary stay.

In a terminology more close to ontology: Why is there not only no fact that Mary leaves, if there is the negative fact of her not-leaving, but also the positive fact of her staying? The answer is that relations between predicates hold not only because of postulates, axioms, definitions, in short – because of logic, but also as a matter of fact. Mutual meaning inclusion (cf. (34) on page 127) may hold simply because of the actual use of the terms in question. The predicates "leave" and "not-stay" on one hand, and "stay" and "not-leave" are synonymous, but not by definition. This approach also defeats Neales second argument against negative predicates based on the requirement of *Veridicality*: If someone sees that α, then α. The truth of (28) must yield that Mary didn't leave. But in a logical representation of (28) we find – he argues – "stay". This is simply not the case, and the following paragraph shows what we find instead.

Consider a predicate-forming operator * that creates a simple negative (contrare) predicate term from a simple positive predicate term. One may accept the following definitions:

(30) A simple predicate term is positive if and only if it is not marked with the symbol *.

(31) P^* is a simple (negative) predicate term if and only if P is a simple positive predicate term.

The operator * is ruled by the following axiom:

(32) $\sim(P(a_1, \ldots, a_n) \wedge P^*(a_1, \ldots, a_n))$

Thus, * is a special contrariety-operator defined on the domain of simple positive predicates, and for every such term it creates a corresponding negative predicate term. This approach implies a considerable generalization of the intuitive notion of a negative property. Namely, we claim that *for every* positive property there is a corresponding negative property. One may doubt such generalization. Many properties, of course, do have negative counterparts: honest – dishonest, or finite – infinite. But what about blue and not-blue (blue*)? Is there really such a property – "not-blue"? The decision in favor of such properties is due to several reasons. First of all, we believe that they do not harm. They are, in fact, much more understandable and tolerable, than all these structured properties suggested by (25) and similar theories. Moreover, we need them for the sake of generality and completeness of the world-description. In this respect Russell's view is of interest, according to which a complete description of the world has to include not only all the positive facts, but also all the negative facts; not only everything that is the case, but also everything that is not the case (see Russell 1992, p. 215). This holds for objects, too. To get a complete characterization of an object, we have to know not only its positive properties, but also all its negative properties; not only what it is, but also what it is not. Often, negative properties of an object are more important than its positive properties. In these cases we even consider such inelegant negative characteristics as "not-blue" to be real properties of the object. Suppose, for example, somebody (X) being in fond of girls with blue eyes only. Consider now a dialogue between Mr. X and his friend Y about a Miss Z:

Y: "Do you like Z?"
X: "No, she hasn't got blue eyes."
Y: "And of what color are her eyes?"
X: "I don't care. I have never noticed. The main point is that her eyes are not blue."

In this context, X used "not blue" as a real (negative) property. And by means of the operator * we are able to generalize this view.

So far, we spoke about properties. In this respect, there is no difference from relations – for every positive simple relation there is a corresponding negative simple relation: "bigger" – "not-bigger", "like" – "dislike". And for a complete description of the world we need to indicate not only who likes whom, but also who dislikes whom.

Our main conclusion is: negative properties and relations form the ontological background for negative facts. Every negative fact is an instantiation of a negative property or relation by objects.

Patterson in (Patterson 1993) also interprets negative facts using the phenomenon of negative properties. Nevertheless he follows Russell in considering negative facts as molecular ones (cf. Patterson 1993, p. 137). Although he emphasizes the difference between external and internal negations, he does not distinguish between them syntactically. Therefore, his notion of negative properties is not quite successful. Considering the sentence "This is not-red", Patterson writes:

> The above proposition says that a certain particular has the property of being not-red. The immediate advantage of this interpretation is that it relieves us of the necessity of saying that 'not' stands for an entity. Instead the word 'not' qualifies the predicate 'red' and the result is the negative predicate 'not-red' which stands for a certain property. Which property? Simply non-redness. Non-redness is that negative property which a particular has when it lacks the property of redness. (Patterson 1993, p. 142)

We do not agree with such a definition of negative properties. E.g., the number 2 lacks the property of redness, but (contrary to Grzegorczyk's

lemon) it does not have the property of not-redness. Negative properties should be defined only with respect to their corresponding positive properties. With respect to objects, negative and positive properties ontologically have the same rights.

A. Fuhrmann analyzing the contrariety-operator sets the following question:

> ...was hat die Konträroperation * mit einer Negation zu tun? Es gibt viele disjunkte Eigenschaften, ohne daß man auf die Idee kommen würde, die eine als Negation der anderen aufzufassen. Tafeln und Stühle, Häuser und Bäume sind disjunkte Prädikate. Was ist besonderes an den Schönen und den Unschönen, den Ehrlichen und den Unehrlichen, daß man sie als Paare auffassen soll, die durch das Band irgendwie verstandener Negation zusammengehalten wird? Ist es die bloße Tatsache, daß die Vorsilben 'un-' oder 'nicht-' verwendet werden, um aus der Bezeichnung der einen Eigenschaft eine Bezeichnung der anderen Eigenschaft zu gewinnen? Tappt man da nicht ins Fliegenglas der Sprache? Scheffelt Scheffler Geld, nur weil er 'Scheffler' heißt? (Fuhrmann 1997, p. 28)

Indeed, we believe that words like "in-", "un-", or "dis-" indicate a certain structure which is connected with negative predicates. There are more of them (in German and in English) than Fuhrmann and we quoted. All of them are natural language equivalents to the predicate forming operator of (30–32). As any other logical operation, a predicate forming negation has certain logical characteristics *due to logical form.* In most cases, such features can be explained in terms of truth-values. Thus, the main feature of the classical propositional (contradictory) negative connective is that the propositions A and $\sim A$ cannot be both true and both false (not true). This holds exclusively due to their form. Some sentences *factually* are in the same relation, too, but no factual information is allowed to be taken into consideration when we analyze *logical* relations between sentences. E.g., the sentences "$2 + 2 = 4$" and "$2 + 2 = 5$" cannot be both true and both false due to the laws of arithmetic, however, the latter is not a logical negation of the former. The main logical feature of properties one of which is a contrare negation of the other is that an object cannot simultaneously have both of these properties, although it can have neither of them, and this is the case *exclusively* due to logical reasons (namely, due to the logical forms of the corresponding predicates). Of course, no thing can be both yellow and blue, but to know this, one has to involve some additional factual information of the nature of this properties, hence the properties "yellow" and "blue" are not contrare negations of each other in a proper sense. On the contrary, "not-yellow" is a contrare negation of "yellow" – these properties enjoy the above mentioned feature that is characteristic for properties which contrarily negate each other. This fact is based on axiom (32), and it is the prefix "not-" alone which settles the matter.

Another important feature of the contrariety operator follows from the above mentioned definitions: It can be applied only to *positive* predicates. Therefore, it is impossible to iterate it – the opposite predicate to P^* is P, but not P^{**}. There is no such property as "disdishonest" – the opposite of "dishonest" is, of course, "honest".

6 Generalization of properties. The universal property

For a better understanding of the phenomenon of negative properties we need to consider further how properties in general interact with objects and with each other. Existence requirements for properties are easily found:

(33) A property exists if and only if there exists at least one object bearing this property.

The definition shows some ontological dependence of the properties on objects. There are no uninstantiated properties. Many properties are inter-related in the sense that if an object possesses a certain property, then it possesses a ramified set of further properties, too. One may call the most interesting case "generalization", because it expresses the idea of finding the "next important larger property". NTP and our theory of negative properties provide us with favorable means for clarifying the idea and for expressing it formally. Let $P < Q$ mean "the property Q is a generalization of the property P". We accept the following definition:

(34) $P < Q$ if and only if $\forall x(P(x) \vee \neg P(x) \longrightarrow Q(x))$.

This definition shows what is the difference between generalization and usual meaning inclusion of predicate terms: To be a proper generalization of P, Q should include not only P itself, but also its negative counterpart. Based on such a generalization, a *hierarchy of properties* arises. Sometimes it is very useful to find a sort of *natural generalization* for a given property P. This should be something like a "natural kind" for P. Without involving ourselves in an obscure metaphysical discussion, we simply suppose that among all possible generalizations of a given property P, we can mark out (if we need) one distinguished property P^{nat} which can serve as a natural generalization of P. Consider the universal property ($\mathcal{E}$) that is instantiated by all the objects in the world:

(35) $\forall x \mathcal{E}(x)$.

The universal property actually is the property of existence – any object first has to exist, and then it can also have other properties. Note that the universal property cannot be generalized, for it is the *largest* property. As

it is known, an attempt to deny its existence can lead to paradoxes and inconsistencies (like "there exists an object that does not exists" etc.). Therefore, in most "theories of existence" some restrictions concerning how to deal with the predicate of existence are formulated. As D. Atlas put it:

> For Russell and Meinong the meaningfulness of negative existential statements necessitates the positing of non-being: (PB) What does not exist must be, or it would be meaningless to deny its existence. (Atlas 1988, p. 375)

Krampitz and Wessel suggested the following axiom for an existence predicate in NTP (reported, e.g., in Krampitz, Scheffler, and Wessel 1996 as a part of a theory of existential charge), in order to overcome these difficulties:

(36) $\sim\neg\mathcal{E}(x)$.

(That is, the predicate of existence cannot be "abgesprochen", denied of an object.)

Why should one accept (36)? The suggested theory of negative properties explains it. In order to find a parallel negative property, $\mathcal{E}$ first must be generalized. Only in-between a natural generalization of the predicate one is allowed to apply the *-operation. Any such natural generalization would immediately lead to a contradiction with respect to our understanding of $\mathcal{E}$ being the most general, the universal property. Consequently, if existence has no natural generalization, it cannot have a negative counterpart, so there is no opposite property for it and we must forbid the negative type of predication for $\mathcal{E}$. This observation justifies the axiom (36).

7 The semantics of negative facts

So far we tried to avoid set-theoretic terminology. We believe that a good theory of properties makes superfluous an ontology based on sets (cf. (Bealer and Mönnich 1989), p. 135). Sets are nothing but mathematical abstractions (or fictions). Nevertheless, some readers could prefer a set-theoretic style of representation, finding it, e.g., more comprehensive from a "methodological" point of view. Especially for those readers we propose a model-theoretic semantics of negative facts.

Let us have a domain of objects – D. Let $\mathcal{P}(D)$ be the set of all the subsets of D, $\mathcal{P}(D^2)$ – the set of all the subsets of the set $D \times D$, $\mathcal{P}(D^n)$ – the set of all the subsets of the set $D \times \ldots \times D$ (n times), etc.

Consider the set $\mathbf{W} = \mathcal{P}(D) \cup \mathcal{P}(D^2) \cup \ldots \cup \mathcal{P}(D^n) \ldots$ By a *model* we mean a structure $\langle \mathcal{P}(\mathbf{W}), h, ng, ^c \rangle$, where

- $\mathcal{P}(\mathbf{W})$ is the set of all the subsets of $\mathbf{W}$ (the set of "possible worlds");
- h is a function of interpretation which ascribes to each subject term a_i an object a_i^h from D, and to each n-placed predicate term a set from $\mathcal{P}(D^n)$;
- ng is a function over elements of $\mathbf{W}$, such that for every $\alpha \in \mathcal{P}(\mathbf{W})$ and for every $P'^n \in \alpha$:

(37) $P'^n \subseteq ng(P'^n)$;

(38) $P'^n \in \alpha \Rightarrow ng(P'^n) \in \alpha$;

- ^{c} is a function that ascribes to every set $ng(P'^n)$ a pair of sets $\langle P^n, P^{*n} \rangle$ ($[ng(P'^n)]^c = \langle P^n, P^{*n} \rangle$) subject to the following conditions:

(39) $P^n \cup P^{*n} \subseteq ng(P'^n)$;

(40) $P^n \cap P^{*n} = \emptyset$;

(41) $(P'^n = P^n) \vee (P'^n = P^{*n})$.

Informally, we follow the usual extensional understanding that identifies properties with sets, n-ary relations with sets of n-tuples. Moreover, any set (say P), containing m elements, represents not only a property P, but also exactly m facts about objects having the property P. The same holds for relations.

The function ng is a function of natural generalization. Function ^{c} forms an opposite property for every property P' within its natural generalization. If P' represents a positive property, the function ^{c} creates the corresponding property P^*, if P' is a negative property, the function creates the corresponding positive property.

Now we are in a position to define the truth-conditions for elementary propositions and logical constants. Let $\|A\|_h^\alpha$ be a value of the formula A in the world α by the interpretation h. Then we have:

(42) $\|P^n(a_1 \ldots a_n)\|_h^\alpha = t \Leftrightarrow P^{n^h} \in \alpha$ and $\langle a_1^h \ldots a_n^h \rangle \in P^{n^h}$;

(43) $\|\neg P^n(a_1 \ldots a_n)\|_h^\alpha = t \Leftrightarrow P^{*n^h} \in \alpha$ and $\langle a_1^h \ldots a_n^h \rangle \in P^{*n^h}$.

Thus, a simple (positive or negative) sentence is true in a world α (by an interpretation h) iff the fact (positive or negative) that makes the sentence true belongs to the world, i.e. the corresponding object(s) have the corresponding (positive or negative) property (are in the corresponding relation).

The truth-conditions for compound sentences can be defined in a usual way:

(44) $\|A \wedge B\|_h^\alpha = t \Leftrightarrow \|A\|_h^\alpha = t$ and $\|B\|_h^\alpha = t$;

(45) $\|A \vee B\|_h^\alpha = t \Leftrightarrow \|A\|_h^\alpha = t$ or $\|B\|_h^\alpha = t$;

(46) $\|\sim A\|_h^\alpha = t \Leftrightarrow \|A\|_h^\alpha \neq t$;

(47) $\|\forall x A(x)\|_h^\alpha = t \Leftrightarrow \|A(a_i)\|_h^\alpha = t$ for every constant a_i;

(48) $\|\exists x A(x)\|_h^\alpha = t \Leftrightarrow \|A(a_i)\|_h^\alpha = t$ for some constant a_i.

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