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# PERSPECTIVES ON TIME

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# PREFACE

The present volume of *Boston Studies in the Philosophy of Science* contains eighteen contributions dealing various aspects of time and written by logicians, physicists and philosophers of science. The authors come from Denmark, Germany, Italy, Poland, Sweden, Ukraine and the U.S.A., where most of them have previously published something on time, either in their mother tongue or in English. The volume opens with a comprehensive essay in which we, the editors, introduce the papers and describe the current situation in the field, tracing some to the historical steps leading up to it. Here we wish to take the opportunity of expressing our gratitude to Professor Bob Cohen for his unfailing interest in philosophy produced outside the English-speaking world and accepting this volume in the Boston Studies series. We would also like to thank Kluwer Academic Publishers, and Annie Kuipers in particular, for advice and assistance during the preparation of the book. Finally, we wish to thank the Thyssen Stiftung and the Netværk for Videnskabshistorie og Videnskabsfilosofi under the Danish Research Council for the Humanities for their generous support of a workshop in 1995 at which some of the authors had the opportunity to meet and discuss their common interest in issues in the philosophy of time.

Copenhagen/Berlin/Toruń

YAROSLAV V. SHRAMKO

# TIME AND NEGATION

## 1. INTRODUCTION

It is not an exaggeration to say that the notion of time is one of the most mysterious. The mysteriousness consists not merely in a vagueness of the notion itself. There are a lot of vague notions, but most of them are at least such that it seems clear within which discipline, science, or branch of knowledge they have to be investigated, i.e. which discipline, science, or branch of knowledge bears the responsibility for the elimination of the vagueness. However, it was repeatedly noted by many researchers that for time the situation is somewhat more intricate. Thus, e.g. G. H. von Wright, defining time as a "fundamental" concept, maintains that it is maybe the best example of a notion which is investigated both by scientists and philosophers. Moreover, he points out that time is simultaneously investigated by several disciplines of natural science (see [15],p. 514-515).

As P. Øhrstrøm and P. Hasle put it:

> people are led into the study of time from a variety of highly different disciplines ([16], p. vii).

A. Anisov in [1] shares the same idea, when he emphasizes that the problem of time

> is among those problems which, being simply characterized as the main subject of scientific research, leaves the question of the specialization of a researcher (whether he is a physicist, or psychologist, or philosopher) open ([1], p. 5)

I might add that there are good reasons according to which an investigator of time could be a philologist, a historian, or someone else[1].

As for logic, time is the subject-matter of a special logical discipline — tense (or temporal) logic. In the present paper, however, I am going

[1] Of course, I am far from considering the problem of time as *the only* problem of this sort. There are a number of other problems which are analogous, among them, e.g. the *problem of man*.

to call in question monopoly of temporal logic in the logical investigation of time. In particular, I will seek to illuminate the interrelations between the notion of time and certain significant notions of some regular (nontemporal) logical systems. In the focus of my attention will be intuitionistic logic and the negation operator. My analysis will not go *too far* into technical details, but it will nevertheless be mainly of a logical character.

## 2. TIME AND CONTRADICTION

In philosophical literature it has been repeatedly pointed out that there exists a certain correlation between the notions of time and of contradiction. Heraclitus was apparently the first who *sensed* this interrelation (although he, perhaps, did not formulate it as a manifest philosophical problem). It is the famous passage

> we do and do not enter in the same river, we exist and do not exist ([B 49a DK]),

which testifies to this fact. There is no explicit reference to time in this passage, nevertheless time can be easily seen to be involved in this context, as well as in many other analogous Heraclitean statements concerning *development* and *becoming*. Indeed, merely the question *why* a river has a capacity to be "such and not such", *what* ensures the very possibility of the existence of these sorts of contradictions, inevitably leads us to a recognition of the key role of time in the processes concerned[2].

Kant paid attention just to this role maintaining that

> Nur in der Zeit können beide kontradiktorisch-entgegengesetzte Bestimmungen in einem Dinge, nämlich nacheinander, anzutreffen sein. ([5], p. 105)

One can also consider the view of A. Schopenhauer, who following Kant, treated time as a form of "innere Anschauung" or as a form of a subject's "Vorstellungen". I would like to emphasize here that Schopenhauer, considering the interrelation between time and contradiction, was even more categorical than Kant was when he claimed:

> Daher kann man die Zeit auch definieren als die Möglichkeit entgegengesetzter Bestimmungen am selben Dinge. ([9], p. 43)

[2] And the analogy itself between time and a river (as a flow which is undergoing constant change) is extremely obvious. In many languages the expression "the river of time" is a common idiom.



By this brief but expressive statement Schopenhauer essentially resolved the problem raised by Heraclitus — about the possibility of a real existence of contradictions in the world. The problem, of course, needs more precise formulation. First of all we have to specify what, as a matter of fact, is meant by the term "a contradiction". Many people (some of them call themselves "philosophers") are prone to treat various couples of different things and metaphorical expressions like "life and death", "the North Pole and the South Pole", "left and right", "black and white" etc. as sorts of contradictions. This leads to a considerable jumble in the understanding of the term in question and only obscures the core of the problem. I would like to express my conviction that the view that *things* may contradict one another is philosophically incorrect. Indeed, consider e.g. a pair "a man and a woman". It is not clear what is meant, when he or she maintains that the elements of this pair contradict one another (if, of course, we consider the question from a philosophical point of view, and do not confine ourselves to the level of everyday common occurrences). One can claim just as well that a man contradicts a tree, and a chessboard contradicts a set of playing-cards. Why, indeed, do black and white contradict one another, but black and blue, or pink and white do not? Therefore, in the present paper I proceed from the point of view that contradictions proper (real contradictions) do appear exclusively on the level of facts, situations, or states of affairs. *A contradiction is a relation between situations, not between things or objects.* Objects can (or cannot) have certain features: as a result we obtain certain situations (facts, states of affairs). And situations, in their turn, *can* contradict one another. Thus it is appropriate to speak about contradictions only in terms of the presence or absence of some situation.

Let $A$ be a situation (fact etc.). Then any contradiction (in the strict sense of the word) can and must be expressed as follows: $A$ obtains and $A$ does not obtain. Consider the pair ⟨the North Pole, the South Pole⟩. The elements of this pair are only some points on the surface of the earth, and as such they do not contradict one another at all; no more than the elements of the pair ⟨the North Pole, Krivoi Rog⟩. Let us examine, however, the following two situations:

(1) This man is at the North Pole.
(2) This man is at the South Pole.

As such these situations do not form any contradiction either. Nevertheless, if we take into account some implied presuppositions, as: the North Pole and the South Pole are *different* points; nobody can simultaneously find him/herself at different points — then it becomes obvious that the situation (1) contradicts situation (2), as well as situation

(3) This man is in Krivoi Rog.

(Please, take into account that Krivoi Rog is neither at the North Pole, nor at the South Pole.) Speaking more precisely, we should say that these situations are incompatible.

It is clear that the only way of resolving such contradictions in a rational manner consists in involving the phenomenon of time and in *the separation of contradictory situations in the course of time.*

Among contemporary philosophers considering this problem one can distinguish G. H. von Wright who, generalizing Kant's approach, claims that if there had been no time the permission of change would have implied the permission of a contradiction. Therefore, from his point of view, if we proceed from the fact that change really happens and is empirically given to us, then we have to describe it as a sequence of contradictory connected states; otherwise we obtain a contradiction. Metaphorically speaking, von Wright concludes, time is a deliverance of man from contradiction (see [15], p. 529).

## 3. TIME AND LOGIC

The view that classical logic completely ignores the temporal characteristics of both the world and of human reasoning is now universally accepted. G. H. von Wright states this fact, too, and points out that (classical) logic traditionally deals with conceptual constructions that apply to a static world. Classical logic considers propositions that invariably are either true or false, and things that definitely either enjoy or do not enjoy the given features (see [15], p. 516). But he also points out that a philosophical concern with human actions on the one hand, and the development of modal logic as well as the consideration of some old philosophical problems (such as the problem of future contingents, raised by Aristotle) on the other hand, caused a logical interest in the notion of time (see [15], pp. 516–517). As a result, we have a special branch of logical analysis (founded by A. Prior): temporal (or tense) logic.



However, both modal and temporal logics directly belong to the sphere of *philosophical logic*. One might think that *mathematical logic* (the logic of mathematics) has no need of such philosophical notions as the notion of time. It seems the originators of modern classical logic adhered to this opinion (let us recall that originally — at the end of the XIX and the beginning of the XX century, when the present form of logic had been established — modern logic had been developed primarily as a logic of mathematics). But the practice of mathematics, like other kinds of human activity, unfolds in the course of time! Therefore, the realization of the timelessness of pure mathematics required some fairly powerful abstractions and idealizations. Among these abstractions is the conception of actual infinity, according to which e.g. the whole infinite set of natural numbers does actually exist as a closed totality. Another implication of the conception of actual infinity is the naive belief that the set of all true mathematical propositions is already determined in some way once and for all (perhaps, there even exists Someone — maybe it is the Lord — who already knows all these propositions). According to this belief, it doesn't matter, if we (unlike the Lord) do not know at some given moment whether one or another concrete mathematical proposition is true or false. This fact can be of some interest only for historians of mathematics, but not for mathematics itself.

Such a belief has been taken for granted practically by all mathematicians, until the appearance of works by L. Brouwer who proposed an alternative view on the very essence of mathematics. (I do not broach here the subject of Brouwer's predecessors — Kronecker and others.) Brouwer's program has been called "intuitionism". Within intuitionism, mathematics is considered to be not just the totality of mathematical propositions, but a certain kind of human activity (an activity of human consciousness) aiming at constructing or "building" (one of Brouwer's favorite words) mathematical objects and propositions. Intuitionism involves both the purely mathematical part (intuitionistic mathematics) and the deeply philosophical considerations (intuitionistic philosophy). The latter is of special interest within this paper.

One of the main philosophical notions upon which intuitionism is built is the notion of the *primordial intuition of time*. This notion, in particular, is the basis of the intuitionistic notion of number (see [10], p. 40). In Brouwer's opinion the primordial intuition of time is a "fundamental phenomenon" of pure mathematics. It is

> the intuition of two-oneness, the primordial intuition of mathematics which immediately creates not only the numbers one and two but all finite ordinal numbers ... (cit. after [10], p. 149)

It also is

> no more than the intuition of time, in which repetition of 'thing-in-time and again thing' is possible ... (cit. after [10], p. 148)

An outcome of such an approach is the treatment of real numbers as a *free-choice sequence* which is infinitely created by the human mind[3]. In other words, the conception of potential infinity is put in the place of the conception of actual infinity.

Thus, historically, intuitionism was the first complete attempt to treat mathematics from a temporal perspective. In what follows I will seek to show that such a treatment has some significant implications for logic itself; the point is that in intuitionism the informal understanding of propositions involves certain temporal aspects, and that the semantic interpretations of some logical connectives (in particular and chiefly — of the negation operator) explicitly involve temporal terms.

At this point I would merely like to note, very briefly, that one of the main features of the intuitionistic view consists in stricter criteria (in comparison with classical logic) for the permissibility of propositions. From an intuitionistic point of view a proposition can be counted as true if and only if it is *constructively proven*. Within intuitionism we deal with *proofs* rather than with *statements*. Obviously the notion of proof itself is connected with the notion of time, for a proof nearly almost is a certain process which is unfolded in the course of time.

## 4. TIME AND MODEL STRUCTURES

As soon as S. Kripke proposed his possible-world-semantics for modal logic ([6]), this approach to semantic analysis immediately acquired a huge popularity within the logic community. This popularity is quite mysterious, for *from a purely technical point of view*, Kripke-semantics does not represent anything new in comparison with algebraic structures which have been known before. Besides, there already existed other effective methods of semantic analysis (e.g. matrixes). However, it was

[3] Sometimes such a treatment is blamed for *mysticism*. However, it may well be that the notion of actual infinity includes much more mystical elements than the notion of a freely creating subject



"the semantics of possible worlds" that arrested the general attention, and now almost every respectable logic calculus should, among other indispensable attributes, enjoy an adequate "Kripke-style" semantics.

Why did just this style of semantic analysis appear to be so attractive? Routley and Meyer ([8]) hold, and maybe they are right, that the reason lies in the field of the "psychology of logic". In this respect it is very important that Kripke- semantics permits a highly natural informal interpretation that provides the possibility of illuminating some deep philosophical foundations of the corresponding logical systems. This fact acquires a paramount significance with respect to the philosophical contents of modal logic. The same holds for tense logic as well.

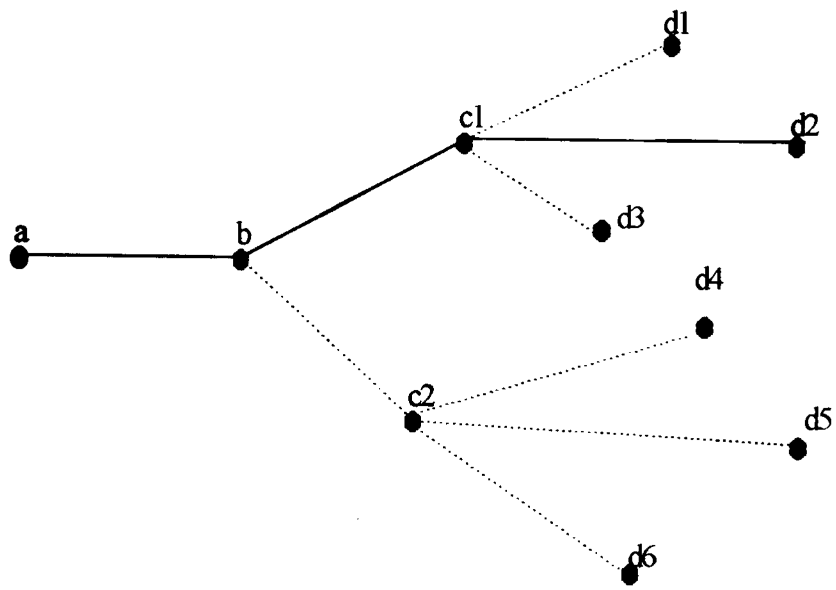
The principal notion of Kripke-semantics is the notion of *model structure*. The main constituents of any model structure are a set of "possible worlds" and an accessibility relation $R$ defined on this set. Of course, the notion of "possible world" itself has a deep philosophical meaning, and the idea of some relationships between worlds permits a considerable extension of possible philosophical interpretations.

Concerning temporal logic, a set of "possible worlds" is usually treated as a set of moments, and relation $R$ as a time-ordering. Then we can, taking different conditions for the relation $R$, simulate various philosophical conceptions of time: branching in the future, linearity, cyclical recurrence etc. However, I would like to emphasize that not only model structures of tense logics can represent certain temporal ideas and have a temporal contents, but the model structures of some nontemporal systems as well.

In [7] Kripke proposed a possible-world-semantics for Heyting's intuitionistic propositional calculus. The intuitionistic model structure (i.m.s.) is a pair $\langle W, R\rangle$, where $W$ is a set of "possible worlds", and $R$ is a transitive and reflexive relation on $W$. Moreover, truth conditions must be formulated with regard for the following *principle of truth-preservation*:

*If a proposition is true in a world $a$ in $W$, then this proposition has to be true in every world $b$ which is accessible from $a$ ($Rab$).*

One of the main points of the present paper is that some regular logical systems, being itself nontemporal on the syntactical level, can nevertheless presuppose and reflect certain deep temporal ideas. These ideas can find their reflection just on the semantical level, namely semantical structures for some nontemporal logical systems can be interpreted as temporal structures. Consider e.g. the following informal interpretation of Kripke's intuitionistic model structures. Every element from a set $W$ can be interpreted as *a state* (a theoretical construction) of a constructive theory *at some moment*. A proposition belongs to such a theory at some moment if this proposition is constructively proven at this moment within this theory. The relation $R$ can be interpreted as a *possible time-relation* between states of the theory. That is, $Rab$ means that the state $a$ of the theory possibly precedes the state $b$ of the theory, in other words, theoretical construction $b$ is a result of some *possible* development of theoretical construction $a$. It is also presupposed that theoretical constructions can undergo no changes at all (reflexivity of $R$!). Thus, the intuitionistic model structure is a picture of the possible developments of our theory in the course of time: the actual line of development is taken into account as well as possible lines which, maybe, will never be actualized. An example of such a picture is presented in the following figure (the uninterrupted line represents the actual development, and the dotted lines stand for possible developments).




The principle of truth-preservation means that an accumulative model of the development of scientific knowledge has been adopted, i.e. theoretical knowledge can only increase — if a certain proposition has been proven, it forever remains proven.



## 5. CONTRADICTION AND NEGATION

I now come to the consideration of the intuitionistic negation operator and its informal interpretation. This interpretation differs radically from that one in classical logic. Let me recall that within the intuitionistic conceptual framework every proposition $A$ has the sense: "$A$ is (constructively) proven". The simple denial of such a statement is: "$A$ is not proven". This is just a classical-style negation, and it is obviously at odds with the "spirit of intuitionism". First of all, the statement "$A$ is not proven" is a simple reflection of the fact that we do not have considered a proof of $A$ *at the moment*. Such a statement cannot be included in a constructive theory because it says nothing about the realm under investigation. It simply is a *metadescription* of a certain state of our knowledge. Such a statement does not satisfy the principle of truth-preservation — at the moment we do not have a proof of $A$, but we may well find one afterwards! Thus, from an intuitionistic point of view, such a statement is of no theoretical interest (although it is, perhaps, of historical interest).

The only intuitionistically acceptable way to deny (to reject) some proposition is to state that it is *refuted*. In other words, a proposition $\sim A$ can be included in some intuitionistic theory if and only if $A$ is refuted.

Consider now, in which way such an understanding finds its reflection in Kripke-semantics for intuitionistic logic. Let $\models$ be a *forcing relation* between worlds and formulas (as a matter of fact $a \models A$ means that $A$ is proven within the theory $a$, or: within the theory $T$ at stage $a$). In the standard Kripke model for intuitionistic logic the forcing relation for negative formulas is defined as follows:

**Definition 5.1** $a \models \sim A \Longleftrightarrow \forall b(Rab \Longrightarrow b \not\models A)$.

Bearing in mind the above mentioned informal interpretation of intuitionistic model structures we should interpret this definition as follows: "$\sim A$ is proven ($A$ is refuted) within the theory $a$ if and only if within any accessible theory $b$ (which is a result of some development of $a$) $A$ is not proven, i.e. *A cannot ever be proven*".

Thus we can see that Kripke's explication of intuitionistic negation operator includes a certain temporal component, and as such is quite natural.

But this explication of the notion of refutability does not reproduce its informal understanding which one can find in intuitionism. From the intuitionistic point of view the statement "$A$ is refuted" ($\sim A$ is proven) means: "if we suppose that $A$ is proven, we obtain a contradiction"! As A. Heyting puts it:

> In mathematical assertions [...] "not" has always the strict meaning. "The proposition $p$ is not true", or "the proposition $p$ is false" means "If we suppose the truth of $p$, we are led to a contradiction. ([3], p. 18)

And then:

> Every mathematical assertion can be expressed in the form "I have effected the construction $A$ in my mind". The mathematical negation of this assertion can be expressed as "I have effected in my mind a construction $B$, which deduces a contradiction from the supposition that the construction $A$ was brought to an end". ([3], p. 19)

Thus we can see that intuitionistic negation and the notion of contradiction are very close to each other. Moreover, informal intuitionistic intuition about negation explicitly involves a significant temporal constituent. In section 1 I already pointed out the interrelation of time and contradiction. Time makes possible the *objective* existence of incompatible situations, namely — their *consecutive* existence which take places at different moments. Language is the sphere where contradictions can have a simultaneous existence. Our theories can be (and often really are) contradictory. And the intuitionistic informal understanding of negation points to contradictions of this sort.

But it also implies a certain temporal aspect: if we wish to refute some proposition $A$, we have to show that any attempt to include $A$ in our theory leads inevitably (in the future) to a contradiction, i.e. any possible development of our theory will result in a contradictory theoretical construction.

To sum up I would like to express my conviction that the definition 5.1 is not quite felicitous, and it could (and ought to) be modified just in accordance with the intuitionistic informal understanding of negation.

Veldman in [14] suggested an interesting modification of the definition of the forcing relation for intuitionistic negation. The main feature of his modification consists in introducing the notion of an *exploded world*: $a$ is an exploded world if and only if $a \models \perp$ ($\perp$ satisfies the condition $\perp \supset A$ for every formula $A$). In Veldman's semantics negation can be defined by



**Definition 5.2** $a \models \sim A \Longleftrightarrow \forall b(Rab \Longrightarrow (b \models A \Longrightarrow b \models \perp))$.

Veldman used his modification for obtaining an intuitionistic proof of the completeness-theorem. Exploded worlds were also investigated in [11], [12], and elsewhere.

Let me interpret an exploded world as a contradictory theoretical construction (a piece of information). Then it becomes clear that Veldman's modification is not only a "technical device" (as Veldman himself writes), but an adequate reproduction of the intuitionistic informal understanding of negation. In what follows I will modify Veldman's approach to defining negation, and this modification will enable me to formulate a general semantics for a collection of intuitionistic systems (up to classical logic)[4]. This semantics will be able to make apparent whether one or another logical system implies a temporal component, and if it does — which conception of time it rests upon. I will also seek to clarify the correlation between some logically valid formulas and one or another philosophical conception of time.

## 6. SEMANTICS FOR CONSTRUCTIVE NEGATIONS

The starting point of further analysis is the positive logic $P$ which is determined by the following axiom schemes and rules:

| | |
|---|---|
| A1. | $A \supset (B \supset A)$ |
| A2. | $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$ |
| A3. | $A \supset (B \supset (A \wedge B))$ |
| A4. | $A \wedge B \supset A$ |
| A5. | $A \wedge B \supset B$ |
| A6. | $A \supset A \vee B$ |
| A7. | $B \supset A \vee B$ |
| A8. | $(A \supset C) \supset ((B \supset C) \supset ((A \vee B) \supset C))$ |

[4] The informal understanding of negation considered above is a general scheme which reflects the general intuitionistic approach to the negation operation. Within this scheme there can coexist various kinds of negation-connectives. A concrete negation-connective of a certain logical system can in some degree or other correspond to that informal understanding. Therefore, just as no concrete logical system can lay claim to the status of being *the only intuitionistic logic* (as Brouwer and Heyting pointed out), so in exactly the same way no concrete negation-connective of a certain logical system can pretend to the status of being the only possible intuitionistic negation. One can only talk about *negations of intuitionistic type* (or constructive negations).

MP. $[A \supset B, A \,/\, B]$.

I will also need the following extra schemes:

PL. $((A \supset B) \supset A) \supset A$
RAA. $(A \supset B) \supset ((A \supset \sim B) \supset \sim A)$
EFQ. $\sim A \supset (A \supset B)$
TND. $A \vee \sim A$
DNE. $\sim\sim A \supset A$.

Curry in [2], ch. 6, considers five systems of *constructive* (as he says) negation. He characterizes the negation of each system by using the terms *refutability* and *absurdity*:

**System** $M = P + RAA$: minimal negation, or simple refutability;
**System** $I = M + EFQ$: intuitionistic negation, or simple absurdity;
**System** $E = M + PL$: classical refutability;
**System** $K = M + DNE$: classical negation, or absolute absurdity.[5]

The system $M$ was proposed by Johansson [4], and also the system $D$. The system $E$ was proposed by Kripke (do not confuse it with the system of relevance logic $E$ (*of entailment*)!).

Thus, if we consider classical negation as the "utmost" case of constructive negation (as Curry does), $M$ — $K$ form a collection of systems for various kinds of constructive negation. The connections between these systems can be described as follows:

$M \subset I$, $I \subset K$, $M \subset D$, $D \subset E$, $E \subset K$, $D \cap I$, $E \cap I$ (where $X \subset Y$ means that the set of theorems of system $X$ is a proper subset of the set of $Y$-theorems, and $X \cap Y$ means that these sets intersect).

One can formulate a world-semantics for positive logic as follows:

A *P-model* is a triple $\langle W, R, \models\rangle$ where $W$ is a non-empty set of "possible worlds" (theoretical constructions), $R$ is a *reflexive* and *transitive* relation on $W$, and $\models$ is a *forcing relation* satisfying the following condition (for $a, b \in W$, and $p_i$ being a propositional variable):

**Condition 6.1** $a \models p_i,\ Rab \Longrightarrow b \models p_i$.

The definition of the forcing relation for compound formulas is as follows:

**Definition 6.2**
$a \models A \wedge B \Longleftrightarrow a \models A$ *and* $a \models B$;

[5]My account of Curry's original consideration is modified in that respect that Curry considers the Gentzen-style L-formulations of the systems, whereas I deal with Hilbert-style axiomatic formulations.



$a \models A \vee B \Longleftrightarrow a \models A$ *or* $a \models B$;
$a \models A \supset B \Longleftrightarrow \forall b(Rab \Longrightarrow (b \models A \Longrightarrow b \models B))$.

$A$ is *verified* in the given $P$-model $\langle W, R, \models\rangle$ (with regard to a given definition of the forcing relation for propositional variables), if and only if $a \models A$ for every $a \in W$.

$A$ is *P-valid* if and only if $A$ is verified in all $P$-models (with regard to every definition of the forcing relation for propositional variables).

Thus a $P$-model is a $P$-model structure plus a certain definition of $\models$ for propositional variables. The informal interpretation of $P$-models is the one developed in section 3. Implication is the positive connective, the semantic definition of which involves the relation $R$. Hence, bearing in mind the informal interpretation of the relation $R$, we can conclude that intuitionistic implication definitely has some temporal characteristics. I will not here consider this situation, because the subject-matter of the present paper is the connective of negation, not of implication. I would like merely to note that from my point of view temporality is and ought to be a quite natural feature of a *real* conditional connective, because an informal understanding of a real implication implies some temporal succession.

**Theorem 6.3** *(For all P-models)*
*For every formula A:* $a \models A$ *and* $Rab \Longrightarrow b \models A$.

**PROOF:** Easy, by induction on $A$.

Now, let me turn to models for negation.

An $M$-model is a 4-tuple $\langle W, R, N, \models\rangle$, where $W, R, \models$ are as above, and $N \subseteq W$ ($N$ can be empty). The definition of the forcing relation for minimal negation is as follows:

**Definition 6.4** $a \models \sim A \Longleftrightarrow \forall b(Rab \Longrightarrow (b \models A \Longrightarrow b \in N))$.

Intuitively $N$ is a set of *contradictory theories* (theory states). We do not take the principle *ex falso quodlibet* as an obligatory condition for elements from $N$, i.e. it may well be that one theory is contradictory in one respect, whereas another (different) theory is contradictory in another respect. Then the definition 6.4 completely corresponds to Curry's informal characterization of minimal negation as simple refutability: a proposition $A$ is counted as simply refuted if and only if the assumption that $A$ belongs to our theory inevitably leads to some contradiction (i.e. our theory will become contradictory in some respect).

System $M$ (including positive logic) can be considered to presuppose and reflect a certain philosophical conception of time: first, time is not linear, but is branching into the future; second, although time itself is a dynamic phenomenon, the temporal behavior of objects can be represented by static models. $M$-models are among these models (as well as $P$-models). Note that an $M$-model is not a model of time itself but a model of the development of our knowledge (our theories) in the course of time. This is why the relation $R$ is reflexive. Of course, time itself strictly implies (at least *some*) change of the world (many researchers demonstrated that if there is no change of the world, then time is impossible). However, our theories are not the entire world — changes can occur in some other places while our theories remain unchanged.

An *I-model* is a 4-tuple $\langle W, R, n, \models\rangle$, where $W, R, \models$ are as above, and $n$ is an element of $W$ which satisfies the following double condition:

**Condition 6.5**
*(1)* $\forall p_i(b \models p_i) \Longrightarrow b = n;$
*(2)* $b = n \Longrightarrow \forall p_i(b \models p_i).$

The forcing relation for $I$-negation (simple *absurdity*) is defined **as** **fol**lows:

**Definition 6.6** $a \models \sim A \Longleftrightarrow \forall b(Rab \Longrightarrow (b \models A \Longrightarrow b = n)).$

**Lemma 6.7** *(For all I-models)*
$\forall b(Rnb \Longrightarrow b = n).$

**PROOF:** Suppose $Rnb$. By 6.5 (2), $\forall p_i(n \models p_i)$. Then by 6.1 $\forall p_i(b \models p_i)$. By 6.5 (1), $b = n$.

**Theorem 6.8** *(For all I-models)*
*For every formula* $A$, $n \models A$.

**PROOF:** Induction by the length of A.

Intuitively, $n$ is *the absolute contradictory* (absurd) theory, i.e. it is the theory where contradictions cannot be localized, because it contains *all* the propositions of the language. Thus, definition 6.6 literally reproduces Curry's characterization of $I$-negation as simple *absurdity*: $\sim A$ is proven (is $I$-refuted) if and only if the assumption of proposition $A$ in our theory will inevitably turn (in the future) this theory into the absolutely contradictory (*absurd*) theoretical construction.

Lemma 6.7 also says that system $I$ implies a conception of the development of knowledge which differs a little from the one of system $M$.



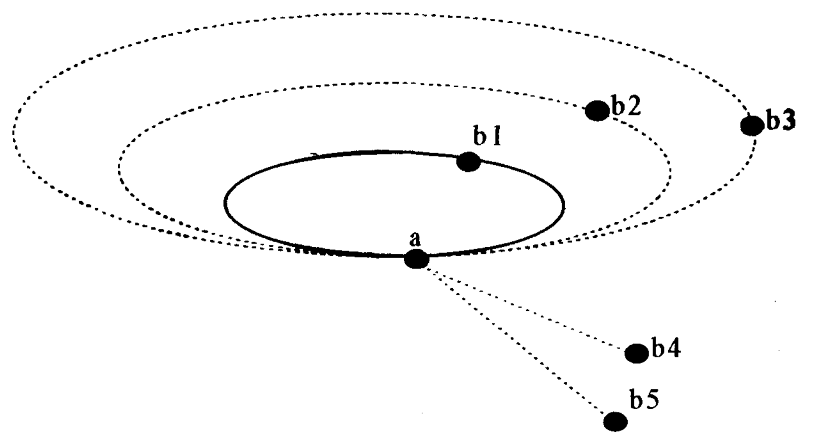
Within system $M$ arriving at a contradictory theory does not mean the cessation of our knowledge development. Within system $I$, on the contrary, when we come (in the course of our theorizing) to a contradiction, the further development of our theory becomes impossible.

A *D-model* is obtained from an $M$-model by adding

**Condition 6.9** *Rab and* $b \notin N \Longrightarrow Rba$.

The definition of the forcing relation for $D$-negation (absolute *refutability*) is the same as for $M$-negation (definition 6.4).

It is apparent that within $D$-models the relation $R$ is not merely reflexive and transitive, but also symmetric *subject to the fact that the accessible world is not contradictory*. This means that by the transition to system $D$ we come to another conception of time. The "philosophy" of system $D$ implies that time is *cyclical* (symmetry of $R$!). In other words, any possible route of our knowledge development will inevitably return to the point of departure. The circle must be closed! However (according to condition 6.9) one or another concrete line of development will return to the initial point only if the chain of theories constituting this line contains no contradictory theories. If we obtain a contradiction on some stage of development, the circle could be broken in this very link. But the branching in the future still remains (see the following figure: $a$ is the given world; $b_1, b_2, b_3$ are noncontradictory worlds; $b_4, b_5$ are contradictory worlds).




The conception of the cyclical recurrence of time is quite interesting from the philosophical point of view, and it can be used in some metaphysical constructions. From a logical point of view, it is very interesting that the logic reflecting such a concept is system $D$.

An interesting question remains: can absolutely contradictory worlds have other uses than their application in the definition of the negation of Heyting's system $I$. The answer is affirmative. The point is that within $I$-models we do not pick out from the set $W$ a set $N$ of simply contradictory theories (because it is presupposed that every such theory would immediately collapse into the absolutely contradictory theory $n$). For $E$-models the situation is different.

An $E$-model is a 5-tuple $\langle W, N, n, R, \models \rangle$, where $W, N, R$ are defined as for $M$-models, and $n$ is an element of $N$, satisfying condition 6.5. Besides the following condition must be held:

**Condition 6.10** $a \neq b$ *and* $Rab \Longrightarrow b = n$.

The $E$-negation is defined by definition 6.4!

**Lemma 6.11** *(For all E-models)*
$a = n$ *and* $Rab \Longrightarrow b = n$.

**PROOF:** Immediately follows from **condition 6.10 and reflexivity of the** relation $R$.

**Lemma 6.12** *(For all E-models)*
$Rab$ *and* $b \neq n \Longrightarrow Rba$

**PROOF:** Assume $Rab$ and $b \neq n$. Suppose $a \neq b$. Then by condition 6.10 $b = n$. A contradiction. Hence $a = b$. By reflexivity of $R$ we obtain $Rba$.

Note that within $E$-models the condition 6.9 becomes provable because of the fact that $n = b \Longrightarrow b \in N$.

Consider now the philosophy of system $E$. Actually, logicians did not pay enough attention to this system (unlike the system $E$ *of entailment*). However, it is of great importance precisely for a temporal characterization of regular logical systems. It is system $E$ which separates two different kinds of logical systems: on the one hand we have logics that rest upon one or another conception of time, and take into account the temporal character of human knowledge, and try to reflect it as a *process*; on the other hand we have logics that ignore all such considerations and which are logics of a "static world". Indeed, the condition 6.10 has the following sense: There is no possibility for any knowledge development, because any attempt to realize such a development immediately turns our theory into the absolute contradictory theoretical construction. For all that the negation operator of system $E$ is still nonclassical, since DNE is not $E$-valid. A $K$-model is obtained from an $I$-model by adding the condition 6.10. The forcing relation for classical negation is determined by the definition 6.6. However, the condition 6.10 nullifies the temporal sense of this definition.



## 7. CONCLUSION

Since I sought to avoid unnecessary technical details (as I promised in the Introduction), the question of completeness and consistency have not been considered in this presentation. There is another paper of mine which is especially devoted to the semantic analysis of constructive negations, and where all the necessary theorems (including those of consistency and completeness) are proved. I would like to list briefly some of the main conclusions of the present paper:

1. The ideas connected with the problem of time can be expressed not only in the framework of temporal logic but also within the semantics of some regular (nontemporal) logical systems.
2. Classical logic is of entirely nontemporal character and is a logic of a static world.
3. Intuitionism is in contrast an attempt to reflect by logical means the temporal character of human knowledge.
4. Both minimal and intuitionistic systems, as well as positive logic, presuppose the conception of time as branching into the future.
5. The law *tertium non datur* ($A \vee {\sim}A$) itself expresses the conception of the cyclical recurrence of time.
6. The key formula which implies the renunciation of temporality is the *Peirce law* — $((A \supset B) \supset A) \supset A$.

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