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State-descriptions as a Method of Semantic Analysis for Intuitionistic Logic

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1. *Classical state-descriptions*

In his book *Meaning and Necessity* Carnap introduced state-descriptions as a method of semantic analysis for classical logic. The method finds its sources in Leibniz's idea of "possible worlds" on the one hand, and some ontological ideas of *Tractatus Logico-Philosophicus* on the other hand. It is well-known that the concept of *fact* is among the principal notions of the *Tractatus*. Facts form a structural basis of the World:

"1 The world is everything that is the case.
1.1 The world is the totality of facts, not of things.
1.11 The world is determined by the facts, and by these being *all* the facts.
1.12 For the totality of facts determines both what is the case, and also all that is not the case.
1.13 The facts in logical space are the world.
1.2 The world divides into facts.
1.21 Any one can either be the case or not be the case, and everything else remain the same."

The precise reflection of this scheme can be found in the logical structure of our language, so that for every fact there is an (atomic) proposition which corresponds to this fact (according to some theories, that *are made true* by this fact).

Carnap's state-descriptions are supposed to be descriptions of the world on the level of facts. Moreover, if we take into account a possibility of alternative developments of the world, we are led to the idea of a set of possible state-descriptions. Now consider a "canonical" definition of state-descriptions:

> "A state-description is a conjunction containing for every atomic statement either it or its negation but not both, and no other statements." (Carnap 1988, p. 224)

A state-description is defined here as a *conjunctive sentence* of the form $^\wedge p_1$ & $^\wedge p_2$ & ... & $^\wedge p_n$ (where $^\wedge p_i$ is a propositional variable or its negation), and this understanding is the most widespread in the logical literature.

It is not entirely clear, however, what is the exact status of state-descriptions in Carnap's theory. On the one hand, Carnap evidently considered state-descriptions among "concepts and methods suitable for semantical analysis" (Carnap 1988, p. 8). On the other hand, state-descriptions explicitly belong to the syntactical level of a language. Therefore Carnap resorts to "reduplication of entities". He introduces the concept of "holding" side by side with the one of "truth". Then he states:

> "a sentence holds in a state-description means ... that it would be true, if the state-description ... were true." (Carnap 1988, p. 9)

One could try to avoid this unwelcome reduplication and directly consider state-descriptions as semantic entities. In this case the definition above would be methodologically incorrect, because by this definition state-descriptions include some elements of object language (e.g., conjunction). The usage of entities of object language as ingredients of semantic constructions violates evidently the demand on division between object language and metalanguage. A definition of the truth-conditions for conjunction would contain then a circle (as the state-descriptions that occur in the right-hand side of the definition include object-language conjunctions).

It is worth mentioning that Carnap proposed also another definition of state-descriptions:

> "A class of sentences ..., which contains for every atomic sentence either this sentence or its negation, but not both, and no other sentences, is called a state-description..." (Carnap 1988, p. 9)

This definition has certain advantages. A state-description is defined here as *a set* of sentences. The object-language conjunction disappears in this way from the state-descriptions – a metalanguage comma is used instead. However, a questionable point remains: the object-language *negation* is still used by a construction of state-descriptions. The above critical remarks concerning conjunction can be reproduced now regarding negation.

Thus, even by the second definition state-descriptions are not fully semantic constructions. If we wish to consider state-descriptions as a pure semantic background (foundation) for some logic, we have to eliminate *all* the logical connectives from them.

2. *Intuitionistic state-descriptions*

In the present paper I would like to propose a new method of semantic analysis for intuitionistic logic that takes into account the criticism above. It would be not enough simply to expel the object language negation from the state-descriptions. According to Wittgenstein's ideas, negation *is* an essential ingredient of the world description on the level of facts. Hence, to be a complete description of the world, a state-description has to contain some kind of negation. Combining this point with the demand to distinguish between a metalanguage and an object language, we are led to the idea of using within state-descriptions some sort of *metalanguage negation*.

Some ideas of Heyting on distinction between "mathematical" and "factual" negation (see especially Heyting 1956, pp. 18-19) can be of use here. Heyting makes a strong difference between these two types of negation. By Heyting *mathematical* negation is the proper intuitionistic negation which can occur in intuitionistic theories. *Factual* negation is a metalanguage negation that is used for describing intuitionistic theories. This *meta*language and its expressions can be unconstructive (do not have to be constructive) – as opposed to the expressions of an intuitionistic *object* language.

Consider some sentence A belonging to an intuitionistic theory. The informal intuitionistic interpretation of the sentence is – "A is (constructively) proved"

(within the theory). The object-language (intuitionistic) negation of such a proposition has to be expressed in the form "A is refuted", or "assertion of A leads to a contradiction". A metalanguage (factual) negation of the proposition is simply "A is not proved".

This distinction between two sorts of negation constitutes the base of the notion of *intuitionistic state-description* which I propose. Let me mark the intuitionistic negation with "~", and metalanguage (factual) negation with "¬". Let $V+$ be the set of propositional variables $\{p_1, p_2, \ldots, p_n, \ldots\}$, and let $V-$ be the set of the propositional variables taken with *metanegations* $\{\neg p_1, \neg p_2, \ldots, \neg p_n, \ldots\}$.

Definition 1.
A set α is an *intuitionistic state description* (i.s.d.) if and only if:
(a) $\alpha \subseteq V+ \cup V-$;
(b) $\alpha \neq \varnothing$;
(c) $p_i \in \alpha$ or $\neg p_i \in \alpha$;
(d) $p_i \notin \alpha$ or $\neg p_i \notin \alpha$.

An *intuitionistic model structure* (*i.m.s.*) is then a pair $< W, R >$, where W is some set of i.s.d. and R is a relation on W defined as follows:

Definition 2.
$R\alpha\beta \Leftrightarrow \alpha^+ \subseteq \beta^+$ [where α^+ (β^-) is the "positive" part of α (β), i.e. α^+ (β^+) is that and only that part of α (β) which consists of the variables without metanegations].

The following lemma can be easily proved:

Lemma 1.
For every i.s.d. $< W, R, >$ and for every α, β, from W:
(1) $p_i \in \alpha$ and $R\alpha\beta \Rightarrow p_i \in \beta$;
(2) $R\alpha\alpha$;
(3) $R\alpha\beta$ and $R\beta\gamma \Rightarrow R\alpha\gamma$.

Let $| A |^\alpha = 1$ mean "A is true in the i.s.d. α", and $| A |^\alpha = 0$ mean "A is false in the i.s.d. α". Then we have the following definition of the truth-conditions for simple and compound sentences:

Definition 3.
$| p_i |^\alpha = 1 \Leftrightarrow p_i \in \alpha$;
$| p_i |^\alpha = 0 \Leftrightarrow \neg p_i \in \alpha$;

$| A \,\&\, B |^\alpha = 1 \Leftrightarrow | A |^\alpha = 1$ and $| B |^\alpha = 1$;
$| A \,\&\, B |^\alpha = 0 \Leftrightarrow | A |^\alpha = 0$ or $| B |^\alpha = 0$;

$| A \vee B |^\alpha = 1 \Leftrightarrow | A |^\alpha = 1$ or $| B |^\alpha = 1$;
$| A \vee B |^\alpha = 0 \Leftrightarrow | A |^\alpha = 0$ and $| B |^\alpha = 0$;

$| A \supset B |^\alpha = 1 \Leftrightarrow \forall\beta\ (R\alpha\beta \Rightarrow (| A |^\beta = 0$ or $| B |^\beta = 1)$;
$| A \supset B |^\alpha = 0 \Leftrightarrow \exists\beta\ (R\alpha\beta$ and $| A |^\beta = 1$ and $| B |^\beta = 0)$;

$| \sim A |^\alpha = 1 \Leftrightarrow \forall\beta\ (R\alpha\beta \Rightarrow | A |^\beta = 0)$;
$| \sim A |^\alpha = 0 \Leftrightarrow \exists\beta\ (R\alpha\beta$ and $| A |^\beta = 1)$;

Thus, we get a special (Kripke-style, cf. Kripke 1965) semantic model for intuitionistic logic on the basis of (or associated with) i.m.s. < W, R >. This model is formulated in terms of intuitionistic state-descriptions. The definitions of validity of a formula in a model and intuitionistically valid formula are standard. The semantics is adequate for the Heyting's intuitionistic propositional calculus **H**.

The proposed semantic construction allows an interesting informal interpretation. Taking into account Heyting's remark that intuitionistic logic is a *logic of knowledge* (as opposed to classical logic which is a logic of being), it is quite natural to interpret an intuitionistic state description as a partial (on the level of atomic sentences) metadescription of a *state of some intuitionistic theory* (at some moment). Then, expression $p_i \in \alpha$ (α is some i.s.d.) means that p_i is proved in the theory which is determined by α (or in the moment α), and $\neg p_i \in \alpha$ means that p_i is not proved in the corresponding theory (or at the corresponding moment). Relation R, can be interpreted as a (possible) time-relation between theoretical constructions. That is, $R\alpha\beta$ means that the theoretical construction β is a result of a possible development of the theoretical construction α.

Another advantage of the approach developed above is that the apparatus of intuitionistic state-descriptions has several important applications. For example, it makes it possible to formulate an intuitive notion of relevant entailment for the formulae of intuitionistic logic, and to provide a natural semantical analysis for some kinds of constructive negations.

3. *Relevant intuitionistic entailment*

Consider the following definition of the consequence relation for intuitionistic propositional logic:

Definition 4.
$A \models B \Leftrightarrow \forall W\ \forall \alpha \in W\ (|A|^{\alpha} = 1 \Rightarrow |B|^{\alpha} = 1)$

This relation is "counterintuitive" and "irrelevant" in the respect, in which the classical consequence relation is. Namely, the following two principles (known as the *paradoxes of relevance*) hold for any intuitionistically valid formula A and for every intuitionistic formula B:

$B \models A$ (*Positive Paradox*)
$\sim A \models B$. (*Negative Paradox*)

One of the most widespread strategy of eliminating these paradoxes is developed within the *relevance logic* (cf. Anderson & Belnap 1975 and Anderson, et al. 1992). In particular, Anderson and Belnap (in Anderson & Belnap 1975, section 15.2) presented a Hilbert-style axiomatic system $\mathbf{E}_{fde}$, which pretended to be a correct formalisation of all valid first-degree *relevant* logical entailments. All the theorems of $\mathbf{E}_{fde}$ are first-degree relevant implications. (A first-degree implication is a formula of the form $A \rightarrow B$, where "A and B can be truth functions of any degree..." (Anderson & Belnap 1975, p. 150). That is, both A and B can contain connectives &, ∨, ~, "but cannot contain any arrows"(*ibd*).)

The Anderson and Belnap's theory of the first-degree entailment has an elaborated semantic basis. The most widely known semantics for $\mathbf{E}_{fde}$ are Dunn's intuitive semantics (see in Anderson, et al. 1992, p. 93) and Belnap's theory of "a useful four-valued logic" for "how a computer should think" (see in Anderson, et al. 1992, p. 506). Sometimes in the literature these semantics are called the "*Semantics on American Plan*". The main idea of the plan is to allow sentences to be both true and false, as well as neither true nor false. The paradoxes of relevance do not hold in such semantics, however the problem of a philosophical justification remains as a critical point of the strategy.

I would like to emphasise, however, that zero-degree formulae A and B in the theory of Anderson and Belnap's are formulae of *classical* propositional logic. Therefore, what the theory is really about, is the relation of relevant logical entailment between formulae of *classical logic*. Had A and B been understood instead as formulae of a different nature, for example, representing constructive propositions of intuitionistic logic, the properties of "$\rightarrow$" themselves would have been rather different. In the last case "$\rightarrow$" would stand for *relevant intuitionistic entailment*.

The notion of intuitionistic state-description shows in which way the strategy along the "*American Plan*" may be employed to obtain such a relation of relevant entailment for formulae of intuitionistic propositional logic. Moreover, on the base of some natural interpretation of i.s.d. it can be shown that in the case of intuitionistic logic the acceptance of the basic principles of this strategy looks philosophically better justified, than in classical logic.

Let me extend (or concretise) the informal understanding of i.s.d. and interpret the expression "$p_i \in \alpha$" not simply as "p_i is proved (in α)", but as "an attempt to prove p_i (in the theory α) was successful". This new interpretation seems to be in closer agreement with a general "spirit of intuitionism", because it has more constructive character. I call this interpretation a *constructive* one, in contrast to the simple classical-style interpretation as described in the previous section. The expression "$\neg p_i \in \alpha$" may be interpreted then as "an attempt to prove p_i (in the theory α) was (appeared to be) unsuccessful". The constructive interpretation uses the circumstance that *before* proving a sentence (and in order to prove it), someone should first *try* to prove it. In other words, it makes the following statement evident: an *attempt* of a proof precedes the proof itself.

Now, under the constructive interpretation, one may doubt about the conditions (c) and (d) of the definition 1. The condition (c) would then imply that an attempt of a proof is being made relative to *any* atomic sentence. This idealisation is obviously too strong, and does not reflect the situation that we have in reality. Surely, many sentences remain out of our theoretical consideration at all. On the other hand, the condition (d) means that if an attempt of proving p_i appears to be successful, then any other attempt of proving p_i should be successful as well, and *vice versa*. This condition can also be omitted in the general case. Thus, we arrive at the idea of a *general state-description*:

Definition 5.
A set α is a *general intuitionistic state description* (*g.i.s.d.*) if and only if:
(a) $\alpha \subseteq V+ \cup V-$;
(b) $\alpha \neq \varnothing$.

This means that a general state-description is an arbitrary (non-empty) subset of the set $V+ \cup V-$. A *general intuitionistic model structure* is a pair < G, R >, where G is a set of g.i.s.d., and R is defined as above. Then we can introduce a relation of the relevant entailment for intuitionistic formulae by means of the following definition:

Definition 6.
$A \models_{rel} B \Leftrightarrow \forall G\ \forall \alpha \in G\ (| A |^{\alpha} = 1 \Rightarrow | B |^{\alpha} = 1)$

This definition introduces an intuitive notion of relevant intuitionistic entailment; it can be easily shown that the paradoxes of relevance do not hold for it.

4. *Semantics for constructive negations*

Consider again an informal understanding of the negation operator in intuitionism. As it was noted above, the negation of a sentence is intuitionistically true if and only if the sentence is disproved (or refuted). This is the case when an assertion of the sentence leads to a contradiction. Such an understanding can be formally modelled in the semantics of intuitionistic state-descriptions.

Return to the classical-style interpretation of intuitionistic state-descriptions from section 2. Even under this interpretation the condition (d) of definition 1 can still be criticised. We have to remember that i.s.d. are descriptions of states of *our theories*. But theories can be (and often really are) contradictory. Thus, if we wish that our semantics reflects the real situation in human knowledge, we have to find some means for describing the inconsistent theories.

Definition 7.
A set α is a *real intuitionistic state description* (*r.i.s.d.*) if and only if:
(a) $\alpha \subseteq V+ \cup V-$;
(b) $\alpha \neq \emptyset$;
(c) $p_i \in \alpha$ or $\neg p_i \in \alpha$.

That is, the real i.s.d. should always be *complete* with respect to metanegation, but can be *inconsistent* with respect to it. This might be seen as a very strange fact: taking into account the underlying intuitive interpretation, this means that a situation can appear, when some sentence is and is not proved simultaneously. How can it be? Does it afford some primary intuitive ideas? I believe, however, that this situation can be explained in an intuitively satisfactory way, even under the classical-style understanding of the expressions "$p_i \in \alpha$" and "$\neg p_i \in \alpha$" as "p_i is proved in α", and "p_i is not proved in α".

Consider the exact meaning of the expression $p_i \in \alpha$. It means that a proof of p_i (in an axiomatic theory α) is given, that is – according to the tradition – there exists a sequence of sentences such that any sentence from the sequence is either an axiom of α, or is obtained by inference rules, and the last sentence of the sequence is p_i. An intuitionistic theory is inconsistent iff there is a sentence A, such that A is proved in it, and $\sim A$ is proved in it. Let us take a theory which is contradictory with respect to p_i. That is, the proofs of p_i and $\sim p_i$ in the theory are given. Hence, there is a sequence of sentences such that any sentence from the sequence is either an

axiom of a theory, or is obtained by an inference rule, and the last sentence of the sequence is p_i, and there is a sequence of sentences such that any sentence from the sequence is either an axiom of a theory, or is obtained by an inference rule, and the last sentence of the sequence is $\sim p_i$. The last observation means, however, that *in fact* p_i is not proved, i.e., the above mentioned "proof" (sequence of sentences) for p_i *proves nothing*. But this "proof" *is still present* (as long as our theory is contradictory)! Thus, we have in some sense a contradictory theoretical situation – formally we have a proof of p_i, but this proof does not prove p_i, so, *actually*, we do not have a proof of p_i. And this happens as it must happen, because our theory is contradictory, and we naturally should expect that the corresponding description of the theory is inconsistent as well. On the level of r.i.s.d. this situation is presented by r.i.s.d. $\{p_i, \neg p_i\}$, and this r.i.s.d. perfectly describes what we have in the case of contradictory theory: we have a "proof" of p_i, but this does not mean *at all* that p_i is "really" proved. In other words, if we have $\{p_i, \neg p_i\}$, it simply means: "Although we have a formal "proof" of p_i, nevertheless, p_i is not true (because the theory, where the "proof" was given, is contradictory)".

Now I am in a position to introduce formally the notion of a *contradictory i.s.d.* α ($con(\alpha)$). This may be done by means of the following definition:

Definition 8.
$con(\alpha) \Leftrightarrow \exists p_i\ (p_i \in \alpha \text{ and } \neg p_i \in \alpha)$.

A pair < C, R > is a *model structure for negation* if and only if C is a set of real i.s.d., and R is a relation on C defined by the definition 2. In the definition of truth conditions for atomic sentences and &, $\vee$, $\supset$ I omit mentioning falsity and deal only with the notion of truth:

Definition 9.
$| p_i |^\alpha = 1 \Leftrightarrow p_i \in \alpha;$
$| A \,\&\, B |^\alpha = 1 \Leftrightarrow | A |^\alpha = 1 \text{ and } | B |^\alpha = 1;$
$| A \vee B |^\alpha = 1 \Leftrightarrow | A |^\alpha = 1 \text{ or } | B |^\alpha = 1;$
$| A \supset B |^\alpha = 1 \Leftrightarrow \forall\beta\ (R\alpha\beta \Rightarrow (| A |^\beta = 1 \Rightarrow | B |^\beta = 1);$

Now one can introduce a new definition of truth-conditions for negation that literally reproduces the informal understanding of the negation in intuitionism:

Definition 10.
$| \sim A |^\alpha = 1 \Leftrightarrow \forall\beta\ (R\alpha\beta \Rightarrow (| A |^\beta = 1 \Rightarrow con(\beta))$.

This definition says that a sentence $\sim A$ is true in a theory α if any attempt to include A into the theory makes this theory inconsistent (contradictory).

By means of this definition the so-called minimal negation is defined, i.e., the negation of the minimal logic **M** of Johansson (see Johansson 1936). It is easy to see that under the definition 10 the formula $\sim A \supset (A \supset B)$ is not valid, whereas $\sim A \supset (A \supset \sim B)$ is. This observation may be seen as a surprise, as we might expect to get the negation of the intuitionistic calculus **H**, but get "only" the minimal negation. It appears that if we wish to have the real Heyting's "mathematical" negation, we have to introduce the notion of *absolute contradictory i.s.d.* α ($abcon(\alpha)$):

Definition 11.
$abcon(\alpha) \Leftrightarrow \forall p_i\ (p_i \in \alpha \text{ and } \neg p_i \in \alpha)$.

To obtain a model structure for the system **H**, one has to add to the model structure for minimal logic the following condition:

Condition 1.
$\forall \alpha \in C\ (con(\alpha) \Rightarrow abcon(\alpha))$.

The definition of the truth condition for the intuitionistic negation is then as follows:

Definition 12.
$| \sim A |^{\alpha} = 1 \Leftrightarrow \forall \beta\ (R\alpha\beta \Rightarrow (| A |^{\beta} = 1 \Rightarrow abcon(\beta))$.

Theorem 1.
For every formula A:
$\forall C\ \forall \alpha \in C\ (abcon(\alpha) \Rightarrow (\forall \beta\ (R\alpha\beta \Rightarrow abcon(\beta)) \Rightarrow | A |^{\beta} = 1))$.

Thus, whereas the minimal logic allows a coexistence of different contradictory theories, this is impossible within the intuitionistic logic. In **M**-model structures two i.s.d. can be contradictory in their different aspects (parts), in **H**-model structures there can only be one contradictory i.s.d. – the absolute contradictory one. This is the difference between *a* contradiction and *the* contradiction which reflects some important intuitive motivations that underlies minimal and intuitionistic logics.

Another interesting system in the vicinity of intuitionistic logic is the system **D** (also introduced in Johansson 1936) which is obtained from the minimal logic by adding as an axiom the formula $A \vee \sim A$. We get a model structure for **D** by adding to model structure for **M** the following condition:

Condition 2.
$\neg p_i \in a$ and $R\alpha\beta \Rightarrow \neg p_i \in \beta$.

Lemma 2.
For all **D**-model structures: $R\alpha\beta$ and not $con(\beta) \Rightarrow Rba$.

Thus, we can see that in model structures for **D** the relation R is not only reflexive and transitive, as for **M** and **H** (lemma 1), but also symmetric *subject to the condition that the accessible world is not contradictory.* The following theorem holds for **D**-model structures as well:

Theorem 2.
For every formula A: $| A |^{\alpha} \neq 1$ and $R\alpha\beta$ and not $con\ (\beta) \Rightarrow\ | A |^{\beta} \neq 1$.

There is an interesting question, whether the absolute contradictory r.i.s.d. can have another usage apart from defining the negation of the Heyting's system **H**. The answer is affirmative. Consider the system **E** (see Curry 1963) which is the system **M** plus the *Pierce Law* $((A \supset B) \supset A) \supset A$. A model structure for **E** is obtained from model structure for **M** by adding

Condition 3.
$\alpha \neq \beta$ and $R\alpha\beta \Rightarrow abcon(\beta)$.

Lemma 3.
For all **E**-model structures: $abcon(\alpha)$ and $R\alpha\beta \Rightarrow abcon(\beta)$.

Lemma 4.
For all **E**-model structures: $R\alpha\beta$ and not $abcon(\beta) \Rightarrow R\beta\alpha$.

Theorem 3.
For all **E**-model structures, for every formula A: $abcon(\alpha) \Rightarrow \mid A \mid^{\alpha} = 1$.

It is also interesting that in this way we can formulate a semantics for the classical logic **K** as a specification of the semantics for the system **E**. **K**-model structure is **E**-model structure plus the condition 1. Another way to get **K**-model structure is **H**-model structure + condition 2.

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